Fractional-Order Digital Filters

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Fractional-order digital filters have developed to provide an alternative solution to higher-order integer-order filters, with increased design flexibility and better performance.

Keywords: fractional calculus ; fractional-order filters ; fractional-order sensors ; fractional-order analog filters ; fractional-order digital filters ; fractional-order applications

1. Introduction

The number of fractional calculus applications has seen a rapid growth over the last decade. Fractional calculus can be easily defined as a generalization of integer-order calculus with the order of the differintegral operators as fractional. Its versatility in modeling and control theory has received a lot of attention recently, although it is still a concept insufficiently understood. This limits the wide acceptance of fractional calculus in industrial use. Fractional calculus has been regarded as a much better way to cover the dynamics of certain type of phenomena, such as anomalous diffusive characteristics [1], viscoelasticity [2], epidemic spreading [3], etc. At the same time, fractional calculus in controller design has increased their flexibility and robustness [4][5]. Review papers dealing with the use of fractional calculus in control engineering have been published recently, such as [3][6][7][8][9].

However, apart from fractional-order models and controllers, the theoretical aspects of fractional calculus have been extended to cover adjacent areas of research, namely sensing and filtering. This has somewhat evolved as a logical step, since actual processes are better modeled using fractional-order systems [10]. At the same time, state estimation is crucial in designing fractional-order controllers [11]. Thus, for a robust state estimation and an efficient noise elimination in fractional-order systems, extensions to a fractional-order of the popular integer-order estimators have been proposed.

It has been widely proven that complex systems can be accurately described by power-law series [12]. For the case of electronic devices, the behavior is given by the sum of various independent actions of the charge carriers, exhibiting the normal distribution. Unknown interactions in the electronic device leads to the second moment of distribution that fails to converge. For the case of real-time sampling, the mean converges rapidly towards infinity, while the standard deviation fluctuates. These systems are best described by the Generalized Law of Large Numbers, resulting in power-law series behavior, with an added -stable component, proving the presence of fractional-order dynamics in any complex system [13][14].

Filters are one of the key elements in the signal processing field. Many filtering techniques have been developed throughout the years for noise reduction, signal modulation, demodulation, amplification, etc. Filters can be analog, consisting of electronic circuits that process the analog signal, or digital, consisting of mathematical filters that process the analog signal after its discretization. The popular field of fractional calculus has also infiltrated into filter design, for both analog and digital cases. For the case of analog filters, the creation of the fractance device, integrating fractional-order dynamics into electronic components such as the fractional-order capacitor has been the starting point of fractional-order analog filters. Fractional-order electronic components are used to create filters that have a larger frequency range and a better response than integer-order filters [15]. However, due to the limitations present in fractional-order physical hardware, there are only a few studies covering the physical realization of fractional-order analog filters, which will be described later in the manuscript.

The applicability of digital fractional-order filters spans on a manifold of domains from data transmission and networking applications [16][17][18], electrical vehicle manufacturing (through the determination of state of charge in lithium-ion batteries [19], aerial vehicle orientation using fractional-order filtering of yaw, pitch and roll signals [20], air-quality assessments through pollution and humidity factors [21], civil engineering targeting the measurement and data processing of various characteristics of buildings such as stiffness and damping [22], different biomedical processes, image processing and many more. Most of the existing implementations of fractional-order filters are in the fields of data transmission and battery estimation, as will be shown in a dedicated section that highlights the benefits of fractional-order filters in real-life applications.

2. Applications of Fractional-Order Filters

The well-known Kalman filter is one of the most popular technique in the field of sensor fusion being employed to compensate the effect of sensor noise [23]. Applications of fractional-order filters cover mostly areas such as data transmission and networking issues, as well as estimations of state of charge in lithium-ion batteries (largely used in several industrial domains, including automotive industry). However, other applications of fractional-order filters cover areas such as aerodynamics, civil engineering, biomedical engineering, etc. This section covers some of the most recent research regarding applications of fractional-order filters.

2.1. Data Transmission and Networking

More practical problems occur when the physical data of a system are measured and analyzed through a network. Therefore, one of the practical areas are communication networks, where effort in analyzing the effect of packet losses has been highly considerable. To this kind of systems, generalization of Kalman filter algorithm can be applied [16][24][25]. For estimation of nonlinear systems, a set of generalized algorithms such as Extended Kalman filter (EKF) and Unscented Kalman filter are given in the literature [26][27][28]; especially, interesting algorithm is the Unscented Kalman filter that, in opposition to the Extended Kalman filter, not required differentiation of nonlinear function. In [27][29], UKF algorithm was used to teaching process of neural networks. In [30], the estimation results for fractional nonlinear systems based on Extended and Unscented Fractional Kalman filter (UFKF) were presented. This subsection offers a revision of some research papers dealing with communication networks.

A fractional-order transmitter in a noisy transmission channel is used in [31]. The transmitter is described as a fractional-order stochastic chaotic system. An extended fractional Kalman filter (EKF) is developed and employed as the received module and a synchronization scheme is designed to be used in cryptography in these systems. Different lemmas and theorems are presented in great detail, along with the proofs. Finally, the equations for the output of the communication channel are derived, which take into account the fact that the transmitter module might have another output that should be encrypted. The encryption/decryption methods are also presented. The proposed technique is tested via a fractional-order stochastic chaotic Chen system. The simulation results validate the theoretical part and show the effective performance of the proposed method in synchronizing fractional-order chaotic systems in the presence of noise.

2.2. Applications Using Lithium-Ion Batteries

One of the applications of fractional-order filters is closely related to the field of electrical vehicles that employ lithium-ion batteries as their main energy source. The reliability of such batteries becomes of increasing importance. Batteries' reliability depends heavily on their Battery Management System (BMS), which determines their State Of Charge (SoC) and State Of Health (SoH). SoC is a good indicator when it comes to mileage prediction, while SoH is a measure of the battery's ability to store and deliver electrical energy. Efficient and non-destructive battery operation in automotive applications requires an accurate SoC estimation by the BMS [19]. As SoC cannot be measured by sensors, an estimation based on an equivalent circuit model of the lithium-ion battery is necessary. Traditionally, the equivalent circuit model consists of an integer-order model. For accurate simulation of the battery terminal voltage, the integer-order model needs a higher order, which causes a significant increase in the number of calculations. Apart from this, research on this topic has shown that many phenomena that occur in these batteries, such as mass transport [32] and the double-layer effect [33], can be well modeled by fractional-order calculus. At the same time, the fractional-order model uses less parameters to achieve higher accuracy [34]. In recent years, fractional-order calculus has been widely applied in battery modeling, from simplified models with fixed orders of differentiation [35] to more complex models with free differentiation orders [36][37][38]. A key drawback is that the order values are obtained using offline methods and do not adapt to changing conditions. A widely used method for estimating SoC based on the equivalent battery model consists of various extension of the Kalman filter.

To improve the BMS' accuracy when it comes to SoH and SoC co-estimation, a fractional-order model is presented in [39]. First, the authors realize a fractional-order equivalent circuit model for the battery. Electrochemical impedance spectroscopy is used to measure the battery response to a multitude of frequencies. The results are used to determine a Nyquist plot that is later employed in the parameter identification procedure that uses global optimization algorithms such as Hybrid Genetic Algorithm and Particle Swarm Optimization (HGAPSO). Additionally, a dual fractional-order extended Kalman filter (DFOEKF) is designed for SoC and SoH estimation. The accuracy of the estimations using DFOEKF is also simulated with different tests. Finally, the battery is physically implemented, and final conclusions are drawn regarding the efficiency of the approach.

The estimation of SoC is also addressed in [40]. In practical implementations, the structure of a lithium-ion battery consists of multiple single battery cells that are connected (either in series or in parallel). To determine the state of each single battery cell, a BMS is employed in each lithium-ion battery. One of the most important parameters the BMS needs to determine is SoC. The authors of [40] propose a simple and feasible equivalent circuit model based on fractional variable-order approach. The estimation of SoC is done by an unscented fractional Kalman filter (UFKF). Its design is described in detail. First, some basic definitions of fractional-order derivatives are introduced, along with the equivalent model of the battery. Electrochemical impedance spectroscopy is used to measure the response of the lithium-ion batteries to different frequencies. A Nyquist plot can be derived based on the measured frequency response, as well as the physical circuit and the equations that describe the behavior of the lithium-ion batteries. These equations are later translated into a state-space model. Next, the equations for the sigma points generation, the state estimation time update, state error covariance time update, output update, state estimation measurement update and the state error covariance measurement update are formulated as well as the initialization of the filter. A dual filter is designed to address the problem of accuracy and quality of estimations. The necessary equations are reformulated, and a block diagram of the dual estimation is presented. Simulations of SoC estimations are then presented. The experimental setup is described and then the hybrid pulse power characterization test is conducted to acquire the offline parameters. After that, the federal urban dynamic schedule and Dynamic Stress Test (DST) are conducted to simulate real driving conditions. The results are promising as the proposed model can accurately describe the behavior of a lithium-ion battery and therefore can produce exact estimation of SoC. A fractional-order model combined with the fractional-order unscented Kalman filter is used in [41] to facilitate SoC estimation.

A study of SoC estimation under different ambient temperatures is performed in [42]. An equivalent circuit model of a lithium iron phosphate battery is established in the form of a first-order fractional model. Different charging and discharging battery capacity tests, as well as open circuit voltage tests were performed. The authors proposed a simplified modeling method considering hysteresis characteristics of open circuit voltage. The parameters of this model were identified at different temperatures based on a particle swarm optimization algorithm with dynamic inertia weight. Finally, the fractional extended Kalman filter was derived. Continuous Dynamic Stress Test conditions were used in the estimation of the battery SoC. The results showed that the estimation method had higher accuracy and increased robustness compared to the integer-order EKF.

2.3. Other Applications

Fractional Kalman filters and their more complex variants are also used in orientation problems in aerodynamics, in biomedical engineering or environmental issues, to name just a few. This subsection highlights some very recent applications of this kind.

Fractional-order complimentary filters are designed in [43] for small unmanned aerial vehicles to handle orientation. Most research papers use Kalman filters for this task and it produces good results when high-quality, high-cost sensors are used. However, in the case of low-cost, low-quality sensors, complementary filters are more adequate, since no assumptions are made with regards to linearity and noise statistics. The concepts of fractional calculus are extended to these types of filters and the results show that the proposed approach is indeed efficient on systems with non-Gaussian. In [23] a fractional Kalman filter (FKF) is implemented for attitude estimation of a moving vehicle. The input signals used are taken from a tri-axial MEMS (Microelectromechanical Systems) inertial sensors, i.e., accelerometer, magnetometer and gyroscope. Sensor fusion is performed on the measurements obtained by these sensors to obtain the vehicle’s roll, pitch and yaw angles.

3. Conclusions

Fractional calculus in modeling and control applications has seen a rapid growth over recent decades. Several physical phenomena have been modeled using fractional calculus tools, while numerous research studies have shown that fractional-order controllers provide for better closed loop dynamics and robustness overall, but are definitely the suitable kind for controlling systems described by fractional-order models. Sensing and estimation is a crucial part for a closed loop system to work efficiently. It was then only a matter of time before several studies on fractional-order sensing and filtering methods emerged.

Fractional-order analog filters have emerged as consequences of the fractance device used for the development of fractional-order electronic components. However, current limitations in fractional-order physical hardware have led to scarce literature regarding the physical realization of fractional-order analog filters. The design of these filters is thoroughly studied and presented in a manifold of research works, mostly from a theoretic perspective. Most of these papers present the physical realization of the proposed analog filter from a conceptual perspective. However, the construction of analog fractional-order filters is limited by the need to create custom electronic components of fractional-order characteristics. The field of analog fractional-order filters will definitely benefit when fractional-order capacitors will be available commercially.

On the other hand, fractional-order digital filters are more abundant and various different approaches have been taken so far. By far, the most popular filtering techniques consist of fractional-order Kalman filters and various extensions such as the fractional-order extended Kalman filter and the fractional-order unscented Kalman filter. About 75% of the featured digital filtering papers focus on this topic, whereas the rest proposes variations of the Butterworth filter and fractional-order delay filters. The fractional-order Kalman filter, fractional-order extended Kalman filter, fractional-order unscented Kalman filter, robust extended fractional Kalman filter and fractional interpolatory cubature Kalman filters are used to deal with nonlinear fractional-order systems. **Table 1** presents an overview of relevant papers associated with digital filtering of complex nonlinear systems.

Table 1. Main fractional-order digital filter papers targeting nonlinear systems.

Title	Year	Reference
Extended and Unscented Filtering Algorithms in Nonlinear Fractional-Order Systems with Uncertain Observations	**2012**	[44]
Dual Estimation of Fractional Variable Order Based on the Unscented Fractional-Order Kalman Filter for Direct and Networked Measurements	**2016**	[30]
State-of-Charge Estimation for Lithium-Ion Batteries Based on a Nonlinear Fractional Model	**2017**	[38]
A Modified Fractional-Order Unscented Kalman Filter for Nonlinear Fractional-Order Systems	**2018**	[45]
A novel cubature statistically linearized Kalman filter for fractional-order nonlinear discrete-time stochastic systems	**2018**	[46]
Nonlinear Fractional-Order Estimator With Guaranteed Robustness and Stability for Lithium-Ion Batteries	**2018**	[47]
Robust extended fractional Kalman filter for nonlinear fractional system with missing measurements	**2018**	[48]
Fractional-order chaotic cryptography in colored noise environment using fractional-order interpolatory cubature Kalman filter	**2019**	[49]
Fractional-order Kalman filters for continuous-time linear and nonlinear fractional-order systems using Tustin generating function	**2019**	[50]
An adaptive unscented Kalman filter for a nonlinear fractional-order system with unknown order	**2020**	[51]
Design of a Robust State Estimator for a Discrete-Time Nonlinear Fractional-Order System With Incomplete Measurements and Stochastic Nonlinearities	**2020**	[52]
Extended Kalman Filters for Continuous-time Nonlinear Fractional-order Systems Involving Correlated and Uncorrelated Process and Measurement Noises	**2020**	[53]
Extended Kalman filters for nonlinear fractional-order systems perturbed by colored noises	**2020**	[11]
Hybrid extended-cubature Kalman filters for nonlinear continuous-time fractional-order systems involving uncorrelated and correlated noises using fractional-order average derivative	**2020**	[17]
Hybrid extended-unscented Kalman filters for continuous-time nonlinear fractional-order systems involving process and measurement noises	**2020**	[54]
Novel hybrid robust fractional interpolatory cubature Kalman filters	**2020**	[55]
Adaptive fractional-order Kalman filters for continuous- time nonlinear fractional-order systems with unknown parameters and fractional orders	**2021**	[56]

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