### Perturbative/Non-Perturbative Quantum Field Theory

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In this review, we focused on the foundations of quantum field theory, which is still believed to be the most fundamental theory, describing in principle all phenomena observed in atomic and particle physics. Unlike quantum mechanics, however, its foundations are still not cleared up. We attempted to describe how some novel approaches lead to a unified picture, in spite of the fact that several difficult open problems remain.

Keywords: quantum fields ; divergent series ; Casimir effect ; Wightman axioms ; singularity hypothesis

#### 1. Introduction

One of the most impressive measurements in physics is of the electron anomalous magnetic moment of the electron, which is known with a precision of a few parts in1014<sup>[1]</sup>. On the other hand, the theoretical prediction needed to match this result requires the use of perturbation theory containing up to five loop contributions. As is well known, higher order contributions in systems containing an infinite number of degrees of freedom lead to the appearance of divergences which are taken care of by renormalization theory <sup>[2]</sup>. Therenormalized perturbation series in enormously successful theories, such as quantum electrodynamics (qed), is, however, strongly conjectured to be adivergentseries (see Reference <sup>[3]</sup> for the first result in this direction), and G. 't Hooft <sup>[4]</sup> has, among others, raised the question whether qed has a meaning in a rigorous mathematical sense.

Several years ago, Arthur S. Wightman, one of the leaders of axiomatic quantum field theory, asked the related question "Should we believe in quantum field theory?", in a still very readable review article <sup>[5]</sup>. After his program of constructing interacting quantum field theories in four space-time dimensions failed, the question remains in the air, and we should like to suggest in the present review that the answer to it does remain affirmative, although the open problems are very difficult.

One of the problems with quantum field theory (qft) is that it seems to have lost contact with its main object of study, viz., explaining the observed phenomena in the theory of elementary particles. One instance of this fact is that, except for a few of the lightest particles, all the remaining ones are unstable, and there exists up to the present time no single mathematically rigorous model of an unstable particle (this is reviewed in Reference <sup>[6]</sup>, and we come back to this point Alternative theories have not supplied any new experimentally measured numbers in elementary particle physics, such as the impressive one in qed mentioned above.

A recent, important field of applications of qft came from condensed matter physics: the structure of graphene, of which certain (possibly experimentally verifiable) phenomena have been successfully studied by Gavrilov, Gitman et al. <sup>[7]</sup> with nonperturbative methods from the theory of strong-field qed with unstable vacuum. So far, however, the only experimental consequence of a non-perturbative quantum field theoretic model concerns the Thirring model <sup>[8]</sup> in its lattice version, the Luttinger model. The latter's first correct solution was due to Lieb and Mattis <sup>[9]</sup>; the rigorous explanation of the thereby new emergent quasi-particles through fluctuation observables was provided by Verbeure and Zagrebnov in Reference <sup>[10]</sup>.

# 2. General Aspects of Nonperturbative Quantum Fields: Wightman Axioms for Interacting Quantum Fields, Dressed Particles in a Charged Sector, and Unstable Particles

We shall try to keep with the general objective of remaining at the theoretical physicist's mathematical level  $\frac{[2][11]}{1}$ . The theory will be defined by its vacuum expectation values (VEV) orn-point Wightman functions of a (for simplicity) scalar fieldA(x):Wn(x1,...,xn)=( $\Omega$ , Because of the previously mentioned singular nature of the quantum fieldsA(x), theA(x)are not (operator-valued) functions of the space-time variablesx=(x0,x →), but ratherfunctionals, denoted byA(f), which may be heuristically pictured as "smeared" objects[dxf(x)A(x), withfsmooth, fast decreasing at infinity functions, taken to belong to the Schwartz space [12]: one speaks of "operator-valued tempered distributions" on the Schwartz spaceS.

In Reference <sup>[13]</sup>, pp. 107–110, it is shown that, if the (n-point)Wightman functionssatisfy(a.)the relativistic transformation law;(b.)the spectral condition;(c.)hermiticity;(d.)local commutativity;(e.)positive-definiteness,

then they are the vacuum expectation values of a field theory satisfying the so-called Wightman axioms, except, eventually, the uniqueness of the vacuum state. We refer to Reference [13] for an account of Wightman theory; also see Reference [14].

We shallassume (a.)–(e.)for then-point functions of the observable fields, with, in addition, the following requirement: (f.)interacting fields are assumed to satisfy the singularity hypothesis (the forthcoming Definition).

In a non-perturbative framework, there exists a theory of renormalization of masses and fields, which "has nothing directly to do with the presence of infinities" (Reference <sup>[2]</sup>, p. 441, Section 10.3). We adopt a related proposal, which we formulate as previously mentioned, for simplicity, for a theory of a self-interacting scalar fieldAof massmsatisfying the Wightman axioms, assumed to be an operator-valued tempered distribution on the Schwartz spaceS.

The following result is the spectral representation of the two-point functionW2(Reference <sup>[2]</sup>, p. 457): (103)W2m(x-y)= $(\Omega,A(x)A(y)\Omega)=1i\int_{\infty}dp(m\circ 2)\Delta+m\circ(x-y)$ , where  $\Omega$  denotes the vacuum vector,  $x=(x0,x \rightarrow)$ , and  $(104)\Delta+m\circ(x)=i2(2\pi)3\int_{\infty}R3d3k \rightarrow e-ix0m\circ 2+k \rightarrow 2+ix \rightarrow k \rightarrow m\circ 2+k \rightarrow 2$  is the two-point function of the free scalar field of massm $\circ$ . It is further assumed that  $(105)(\Omega,A(f)\Omega)=0\forall f\in S$ . The spectral measurepmay be further decomposed in the sum of a discrete and a continuous part  $(106)dp(m\circ 2)=Z\delta(m\circ 2-m2)+d\sigma(m\circ 2)$ , where  $\sigma$  is a continuous measure and

Of course, ifZ=0in (106), there is no discrete component of massmin the total mass spectrum of the theory. The positivity ofZis due to the positive-definiteness conditions e.) See Reference [11], p. 50, or (Reference [2],

One of the most important features of relativistic quantum field theory is the behavior of the theory at large momenta (or large energies). Renormalization group theory <sup>[15]</sup> has contributed a significant lore to this issue (even if none of it has been made entirely rigorous): it strongly suggests that the light-cone singularity of the two-point functions of interacting theories is stronger than that of a free theory: this is expected even in asymptotically free quantum chromodynamics, where the critical exponents are anomalous. We refer to this as the "singularity hypothesis", which will be precisely stated in the next section. This hypothesis is also verified in each order of perturbation theory (if the interaction density has engineering dimension larger than 2).

In order to formulate the singularity hypothesis in rigorous terms, the Steinmann scaling degreesdof a distribution  $[\underline{16}]$  is a natural concept: for a distribution  $\subseteq S'(Rn)$ , letu $\lambda$  denote the "scaled distribution", defined by  $u\lambda(f) \equiv u(f(\lambda-1\cdot))$ . As $\lambda \rightarrow 0$ , we expect that  $u\lambda \approx \lambda - \omega$  for some $\omega$ , the "degree of singularity" of the distributionu. (108)sd(u) $\equiv$ inf $\omega \in R | \lim \lambda \rightarrow 0 \lambda \omega u \lambda = 0$ , with the proviso that, if there is nousatisfying the limiting condition above, we setsd(u)= $\infty$ . For the free scalar field of massm $\geq 0$ , it is straightforward to show from the explicit form of the two-point function in terms of modified Bessel functions that (109)sd( $\Delta$ +)=2 (see, e.g., Reference  $[\underline{11}]$ , (5.15)).

We say that thesingularity hypothesisholds for an interacting scalar field if (110)sd(W+)>2.

In Reference  $\left[\frac{17}{2}\right]$ , it was proved that:

If the total spectral mass is finite, i.e., $(111)\int 0 \propto d\rho(a2) < \infty$ , then $(112)sd(W+) \le 2$ , i.e., the scaling degree of W+cannot be strictly greater than that of a free theory; thus, by Definition 1, the singularity hypothesis(110) is not satisfied.

The singularity hypothesis holds for an interacting scalar field only if  $\int 0 \cos(m \circ 2) = \infty$ . This necessary condition is independent of the value of  $0 \le Z \le \infty$ .

The importance of the above theorem, and especially of its corollary is that it provides a mathematical foundation for the forthcoming interpretation of the condition (113)Z=0. In this sense, it is a complement to the foundations of quantum field theory. In order to understand why this is so, we have to make a brief interlude.

For the purposes of identification with Lagrangian field theory, one may equate theA(.)of (103) with the "bare" scalar field $\phi$ B(Reference <sup>[2]</sup>, p. 439), whereby (114)A=ZAphys, under the condition (115)Z>0. Under the same condition (115), the assumption of equal time commutation relations (ETCR) for the physical fields may be written (in the distributional sense) (116) $\partial$ Aphys(x0,x  $\rightarrow$ ) $\partial$ x0,Aphys(x0,y  $\rightarrow$ )=-iZ $\delta$ (x  $\rightarrow$  -y  $\rightarrow$ ). Together with (103) and (116), one obtains (Reference <sup>[11]</sup>, (9.19), Reference <sup>[2]</sup>, (10.7.18) (suitably modified by the factor1Z)):(117)1Z=f0 $\propto$ dp(m  $\circ$ 2).

Formula (117) has been extensively used as a heuristic guide, even, for instance, by the great founders of axiomatic (or general) quantum field theory, Wightman and Haag. Indeed, in Reference <sup>[18]</sup>, p. 201, it is observed that " $\int 0 \infty d\rho (m \circ 2) = \infty$  is what is usually meant by the statement that the field-strength renormalization is infinite". This means that the fields in an

interacting theory are more singular objects than in the free theory, and we do not have the ETCR." Both assertions seems to substantiate the conjecture thatZ=0is expected to be ageneralcondition for interacting fields.

If (117) holds, onlyZ=0is compatible with the singularity hypothesis.

It follows from Corollarys 1 and 2 that the two definitions ofZ, in (106) and in (117), arenotequivalent. Since the ETCR, which implies (117), is not generally valid for interacting fields, as briefly reviewed in the forthcoming paragraph, we conclude that the singularity hypothesis opens the possibility of the non-universal validity of (113).

The hypothesis of ETCR has been in serious doubt for a long time, see, e.g., the remarks in Reference [13], p. 101. Its validity has been tested [19] in a large class of models in two-dimensional space-time, where it was definitely proved not to hold in the case of the Thirring model <sup>[8]</sup> for large coupling. The only case found in [19] in which the ETCR holds was the Schwinger model, which is known to be equivalent to a canonical theory of a massive vector field! A nother interesting case of canonical behavior were the canonical interacting quantum fields on two-dimensional de Sitter space <sup>[20]</sup>, where, however, the canonical character was intrinsically due to the geometry of de Sitter space.

Unfortunately, however, the ETCR (for interacting fields!) is still assumed to hold, without comment, in standard treatises (<sup>[2]</sup>, p. 460; <sup>[11]</sup>, p.47).

Although, when0<z<∞,zis interpreted="" as="" the="" non-perturbative="" field="" strength="" renormalization,="" relating="" "bare"="" fields="" to="" physical="" fields,="" in="" (114),="" remaining=" case="" (113)="" remains="" be="" understood.="" stated="" reference="" [<a="" class="html-bib" href="https://www.mdpi.com/2218-1997/7/7/229/htm#B2-universe-07-00229">2], p. 461, "the limitZ=0has an interesting interpretation as a condition for a particle to be composite rather than elementary". This brings us to our next topic. </z<∞,zis>

#### 3. A Proposal for the Meaning of the Condition Z=0: The Presence of Massless and Unstable Particles

In the presence of massless particles (photons), Buchholz <sup>[21]</sup> used Gauss' law to show that the discrete spectrum of the mass operator (118)P $\sigma$ P $\sigma$ =M2= Above,POis the generator of time translations in the physical representation, i.e., the physical hamiltonianH, andP  $\rightarrow$  is the physical momentum. This fact is interpreted as a confirmation of the phenomenon that particles carrying an electric charge are accompanied by clouds of soft photons.

Buchholz formulates adequate assumptions which must be valid in order that one may determine the electric charge of a physical state  $\Phi$  with the help of Gauss' law:(119)( $\Phi$ ,j $\mu\Phi$ )=( $\Phi$ , $\partial\nu$ F $\nu$ ,  $\mu\Phi$ ). F $\nu$ , $\mu$ denotes the electromagnetic field observable, and (119) is assumed to hold in the sense of distributions onS(Rn).

When endeavoring to apply Buchholz's theorem to concrete models, such asqed1+3, problems similar to those occurring in connection with the charge superselection rule <sup>[22]</sup> arise. The most obvious one is that Gauss' law (119) is only expected to be valid (as an operator equation in the distributional sense) in non-covariant gauges, the Coulomb gauge in the case ofqed1+3, but not in covariant gauges <sup>[22]</sup>. If we adopt the present framework, our option is to use the Coulomb gauge and to define the theory in terms of then-point Wightman functions of observable fields, i.e., gauge-invariant fields, thus maintaining Hilbert-space positivity.  $\beta \int 0 \infty d\rho 2(m \circ 2) \Delta + (x-y;m \circ 2)$ , withdpph,dp1,dp2positive, measures, andp1satisfying certain bounds with respect

Forqed1+3in the Coulomb gauge, assuming it exists in the sense of the framework of this section and satisfies the assumptions of Buchholz's theorem, the following condition holds:

#### Z 2= 0 (122)

Above,Ψdenote observable fermion fields, which we assume to exist as a generalization of those constructed by Lowenstein and Swieca <sup>[23]</sup> inqed1+1; also see Reference <sup>[24]</sup> for a similar attempt in perturbativeqed1+3. In the words of Lieb and Loss <sup>[25]</sup>, who were the first to observe this phenomenon in a relativistic model of qed, "the electron Hilbert space is linked to the photon Hilbert space in an inextricable way". Thereby, in this way, "dressed photons" and "dressed electrons" arise as new entities.

We now come back to Weinberg's suggestion that the conditionZ=0describes unstable particles.

Turning to scalar fields for simplicity, we consider the case of a scalar particleC, of massmC, which may decay into a set of two (for simplicity) stable particles, each of massm. We have energy conservation in the rest frame andq $\rightarrow$ ithe momenta of the two particles in the rest frame ofC:(123)mC>2m. In order to stress the practical effects of the confusion of

the two definitions of Zin the literature, when the ETCR is assumed (including <sup>[2]</sup>), it should be remarked that the first reference given by Weinberg on a model for unstable particles <sup>[26]</sup> assumes (what ammounts to)mC $\leq$ 2minstead of (123)

In order to check that Z=ZC=0 when (123) holds, while  $0 < z < \infty$  is valid=""" in="" the="" stable="" casemc<2m,="" a="" model,="" we="" are="" beset="" with="" difficulty="" to="" obtain="" information="" on=" two-point=" function.=" <="" p=""></z<\infty

There exists a quantum model of Lee type of a composite (unstable) particle, satisfying (123), where (113) was indeed found, that of Araki et al. [27]. Unfortunately, however, the (heuristic) results in Reference [27] have one major defect: their model contains "ghosts". A very good review of the existent (nonrigorous) results on unstable particles is the article by Landsman [28], to which we refer for further references and hints on the intuition behind the criterion (113).

In the next section, we come back to a set of models for atomic resonances and particles, which might support the suggested picture of quantum field theory in terms of "dressed" and unstable particles, and, in the last section, we discuss the crucial conceptual issues and difficulties associated with this program, comparing it with alternative approaches.

## 4. Models for Atomic Resonances, Unstable and "Dressed" Particles: What Distinguishes Quantum Field Theory from Many-Body Systems?

In Reference  $[\underline{G}]$  the model below—the Lee-Friedrichs model of atomic resonances—was revisited. onL2(R3)(see, e.g., Reference  $[\underline{29}]$ ), which describes the photons. Then, the one-dimensional subspaceP0Hconsists of the ground state vector  $(132)\Phi0\equiv|-)\otimes|\Omega)$ , with energy zero, where  $(133)\sigma z|\pm)=\pm|\pm)$  denote the upper|+)and lower|-)atomic levels, and| $\Omega$ )denotes the zero-photon state inF. Note that $\Phi$ 0is also eigenstate of the free HamiltonianH0, with energy zero, and we say, therefore, that the model has a persistent zero particle state.

We shall refer to the model described by (124) as Model 1. with the further correspondences  $|+)=V+|0\rangle,|-)=N+|0\rangle$ , where [0)denotes the fermion no-particle state, Model 1 becomes the usual Lee model for particles, and Model 2 a "refined" Lee model for particles; we refer to them as Model 3 and Model 4, respectively. There is one big difference, however, between the atomic resonance case and the particle case. To the particle versions, Model 3 and Model 4, assuming they are well-defined, the forthcoming framework is applicable.

Let a theory of a scalar field of massm>0be invariant under the euclidean group, that is, the group of translations and rotations of euclidean spacex  $\rightarrow Rx \rightarrow +a \rightarrow$ , where R denotes a rotation. By Haag's theorem (Reference <sup>[18]</sup>, p. 249), in a euclidean field theory which uses the Fock representation, the no-particle state  $\Psi$ 0 is euclidean invariant, i.e., (135)U(a  $\rightarrow$ , R) $\Psi$ 0= $\Psi$ 0. We have the following:

Let the Hamiltonian be of the form(136)H= $\int H(x \rightarrow) dx \rightarrow$ , where  $H(x \rightarrow)$  satisfies(137)U( $a \rightarrow$ , 1) $H(x \rightarrow U(a \rightarrow, 1)-1=H(x \rightarrow +a \rightarrow)$ . Then, if  $\Psi$  is any state invariant under  $U(a \rightarrow, 1)$ , i.e., (138)U( $a \rightarrow, 1$ ) $\Psi=\Psi$ , then  $\Psi$  belongs to the domain of H only if  $H\Psi=0$ .

We have that  $(\Psi, H(x \rightarrow)H(y \rightarrow)\Psi)$  depends only on  $x \rightarrow -y \rightarrow$ , so that  $(139)||H\Psi||2=\iint dx \rightarrow dy \rightarrow (\Psi, H(x \rightarrow)H(y \rightarrow)\Psi)=0$  or  $\infty$ , according to whether  $H\Psi=0$  or  $||H\Psi||=\infty$ , that is, Hmust annihilate any translation-invariant state to which it is applicable.  $\Box$ 

This result may be applied to Model 3 (assuming the renormalizations performed such that the pointwise limit can be taken, leading to an euclidean invariant quantum field theory), with the no-particle state identified to the state given by $\Psi \equiv |\Omega) \otimes |-\rangle$ , in and  $|\Omega\rangle$  the no-particle photon state: We say that there is novacuum polarization. On the other hand, for Model 4,  $\Psi$  does not belong to the domain of Hdue to the term $\sigma$ +a † (g)in (134), which, in terms of fermion operators equals, V+Na†(g), andV+N|-)=|+\rangle. In the case there is vacuum polarization, Theorem 3 implies that there exists an "infinite energy barrier" between the Fock no-particle state and the true vacuum: non-Fock representations are required,

All this being said, Model 3 turns out, very unexpectedly, to be afflicted by "ghosts", i.e., states of negative norm (see Reference <sup>[11]</sup>, Chapter 12). We say unexpectedly because the occurrence of "ghosts" in relativistic quantum field theory is known to be a consequence of the use of manifestly covariant gauges (see Reference <sup>[15]</sup>), and the Lee model is not relativistically invariant. One may consider, however, the model with recoil, describing the interaction between the photon field andtwoparticlesVandN, with energiesEV(p)=(M2+p2)1/2andEN(p)=(m2+p2)1/2, and interaction energyHI=fd3pd3kg(p,k)[V † (p)N(p-k)a(k)+h.c.]\lambda, with $\lambda$ proportional to the charge, andg(p,k)=f(p,k)[8EV(p)EN(p-k)\omega(k)] -1/2,\omega(k)=(\mu2+k2)1/2, the latter representing the (eventually massive) "photon" energy.

We henceforth refer to the Yndurain versions of models 3 and 4 as Model 3Y and Model 4Y.

Model 2 (for zero temperature, as a model of atomic resonances) may be the simplest prototype of a model with vacuum polarization. Model 3Y should be suitable to study the criterionZ=0for unstable particles. The subtlest point in this connection is the fact that the term proportional to the delta measure in (106) has coefficientZ, and, in its absence, due to the conditionZ=0, the (renormalized) mass seems to remain undetermined. One must, therefore, be able to determine the renormalized mass uniquely fromalternativegeneral conditions on the Hamiltonian, such as the requirement that it be self-adjoint and bounded below.

We close this section with some remarks of what distinguishes quantum field theory from many-body systems. In contrast, quantum field theories display in general vacuum polarization, as typified by Model 4Y, which is an approximation of the usual trilinear coupling terms which occur in qed and also in quantum chromodynamics (qcd), as a consequence of relativistic invariance and the gauge principle (see References <sup>[2][15]</sup>). Coupled with Haag's theorem and Theorem 3, this implies non-Fock representations, as previously discussed. What we wish to emphasize is that, in the case of quantum field theories, such representations arise due to a particular reason, namely vacuum polarization.

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