SU(2)/SU(3) Quantum Yang-Mills theory Thermodynamics

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An outline of the main, purely theoretical ideas involved in Quantum Yang-Mills thermodynamics is given and implications thereof for applications in cosmology, particle, plasma, and condensed-matter physics are sketched. On the theoretical side, we elucidate the concepts of the thermal ground states of the deconfining and preconfining phases together with their gauge-mode excitations , and we discuss the quantum vacuum of the confining phase including its finite-extent excitations. On the application side, we briefly mention how deconfining SU(2) Yang-Mills thermodynamics, when postulated to describe thermal photon gases, predicts a modified temperature (T) -redshift (z) relation for the Cosmic Microwave Background (CMB) which, in turn, implies a rearrangement of the dark sector well before the onset of nonlinear structure formation. All-z fits of the ensuing cosmological model to the observed angular power spectra (CMB) yield a value for the present Hubble parameter H_0 agreeing with that extracted from local distance measurements, a baryon density of the present Universe being about 30% smaller than the standard value obtained from Big-Bang-Nucleosynthesis (BBN) but matching direct censuses, and a late onset of reionisation of the Universe agreeing with the observation of the Gunn-Peterson trough in high-z quasar spectra. We also mention how the three lepton families of the Standard Model of Particle Physics (SMPP) could emerge as solitons immersed into the confining phases of three SU(2) Yang-Mills theories, subject to mixing of their Cartan subalgebras. In particular, the electron and its neutrino would be represented by 1-fold selfintersecting and single, stable center-vortex loops with a wealth of implications for strongly correlated charge carriers in the two spatial dimensions of certain condensed-matter systems as well as ultra hot plasmas.

Keywords: thermal ground state ; adjoint Higgs mechanism ; caloron center ; quantum of action ; cosmological model ; value of H0 ; strongly correlated electrons in 2D ; cosmic neutrinos ; baryon density

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1. Introduction

For simplicity we restrict ourselves to the gauge group SU(2). (SU(3) is only quantitatively more complex.) If we may, motivated either by perturbation theory's asymptotic freedom[1][2] or selfconsistently as it turns out [3], assume the existence of a mass scale, originated by quantum dynamics, in enriching the classically defined, conformal theory in four Euclidean spacetime dimensions then the regime of temperatures larger then this mass scale is called deconfining phase. In contrast to the approach, which assumes the expandability of all relevant thermodynamical quantities into powers of a small coupling – perturbation theory, deconfining Yang-Mills quantum thermodynamics really possesses a nontrivial a priori ground-state structure [3]. Spatially, this thermal ground state can be understood in terms of densely packed centers of calorons and anticalorons [4][5][6][7], which give rise to the quantum nature of the theory [HB2016], and their overlapping peripheries. (Anti)calorons are periodic-in-time, (anti)selfdual gauge-field configurationsof topological charge-modulus unity, which, in their [8]trivial-holonomy form, are constructed from a superposition of infinitely many (anti)instanton prepotentials (singular gauge) stacked along the time axis. Shifting at spatial infinity the four-component A_4 of these configurations from zero to certain finite values (nontrivial holonomy) respects their (anti)selfduality yet introduces a discernible magnetic dipole structure (w.r.t. the U(1) Cartan subalgebra singled out by the Lie-algebra valued A_4) which, however, is instable under perturbations of this configuration [9]. Typically, this structure then collapses to a magnetic monopole being infinitely smeared and massless and its antimonopole becoming pointlike. For values of the holonomy far from trivial, however, monopole and antimonopole materialise by their mutual repulsion [9]. They are long-term constituents of the deconfining plasma, subject to magnetic screening by transient dipoles associated with (anti)caloron holonomies close to trivial, and to be interpreted in an electric-magnetically dual way once the Cartan algebra of the theory is postulated to describe electromagnetic disturbances [8]. The radial separation between a(n) (anti) caloron center and its periphery is set by a temperature-dependent length scale emerging from a spatial average over a field-strength two-point correlator, associated with the center, which introduces a fixed mass scale into the theory. (Anti)caloron peripheries exhibit selfdual dipole moments which are directed parallel to a externally applied electric or magnetic field of sufficiently low temporal frequency. Subjecting these dipole moments to a packing density set by the spatial size of (anti)caloron centers, one derives that the propagation speed due to undulating dipole densities of such electromagnetic disturbances (waves) is *independent* of temperature. For a given SU(2) theory, however, only modes of frequencies low than a certain T-dependent maximum value, can propagate as classical waves. It turns out that the observation of electromagnetic *waves* up to the hard X-ray regime enforces a mixing of the Cartan subalgebras of a least two SU(2) theories of disparate quantum mass scales. Excitations of a given theory that associate with higher frequencies probe (anti)caloron centers and therefore are subject to quantum behaviour (the action of a(n) (anti)caloron turns out to be given by Planck's quantum of action [8]):Their energy-momentum dispersion either is that of a massless particle (photon) or, by virtue of the adjoint Higgs mechanism, exhibits a temperature dependent mass gap [10]. Quantum effects that involve more than one (anti)caloron center (overlap of centers and packing voids) are computable in terms of a nonperturbative loop expansion which renders them well-controlled and small corrections to the free-field behaviour [11][12][13].

A thin, preconfing phase is characterised by a superconducting thermal ground state, composed of massless and condensed (anti)monopoles, and Meissner-massive Cartan excitations (off-Cartan excitations decouple at a critical temperature comparable to the quantum mass scale). At a slightly lower temperature, these remaining gauge modes decouple, and since the entropy density would then vanish at a finite temperature (in contradiction to Nernst's third law of thermodynamics) this situation must no longer be subject to thermodynamics: The theory undergoes a nonthermal (Hagedorn) phase transition, characterised by an over-exponentially-in-energy rising density of states which is carried by n-fold selfintersecting center-vortex loops of a multiplicity roughly growing like n. Stable excitations in the low-T confining phase are the single and 1-fold selfintersecting loops which, for a given value of the quantum mass scale of an SU(2) theory, could associate with the corresponding lepton doublet of the SMPP. The energy density of the confining ground state, composed of massless, single round-point center-vortex loops is precisely zero[14]. Due to separated topological charge centers in the deconfining phase an axion field, which arises from a chiral symmetry breaking (massless fermions with universally nonzero gauge couplings) that occurs at temperatures much larger than the quantum mass scale of any given SU(2) theory associated with particle-physics mass scales, becomes massive by the axial U(1) anomaly which has cosmological consequences (dark sector).

2. theory

If once postulates thermal photon gases to be described by an SU(2) rather than U(1) gauge principle then the quantum mass scale ~ 10^(-4) eVof the former theory can be extracted from the observation of an excess in line temperature at low frequencies in the CMB [15]. This theory implies at modified T-z relation which, at sufficiently high T>T_0 (T_0 the present CMB temperature of 2.725 K), is linear and characterised by the critical correlation-length exponent of the 3D Ising transition (no magnetic field) [16]. At low z there is curvature in this function due to the onset of conformal breaking due to the quantum mass scale of the SU(2) theory. The new (flatter-than-standard) T-z relation implies a higher photon-decoupling redshift at T~ 3000 K and therefore less dark matter then. For the cosmological model to incorporrate the successes of the standard Lambda-Cold-Dark-Matter model (LCDM) at low redshifts one therefore is forced to include a release of dark matter from dark energy somewhere in the dark ages as the Universe cools down. Standard computations of the CMB angular power spectra in the new model reveal that the tension in the value of H_0
(local vs global cosmology) is resolved in favour of the local value. Moreover, the high redshift of about 10 for reionisation, extracted from LCDM fits to the observed CMB angular power spectra [17], is brought down to about 6 as extracted from the detection of the Gunn-Peterson trough in high-z quasar spectra [18]. Assuming adiabatic, primordial curvature perturbation their spectrum turns out to tilted towards the red (and not scale invariant as in LCDM), however, and the present baryonic density comes out to be about 30% smaller than the standard BBN value. There is a good chance that radiative effects at low z induce a dynamical breaking of statistical isotropy which would introduce a local depression in CMB temperature around our vantage point thus explaining a deviation from Gaussianity around the CMB cold spot and other large-angle anomalies [19]. The new model for the thermal photon physics paradoxically also fixes the dark sector once we assume that dark matter and dark energy refer to depercolated and percolated (selfgravitating) solitons of the axion field mentioned above.

The presence of a Hagedorn transition is relevant for an understanding of plasma instabilities once the electron and its neutrino are subjected to the two stable, nonlocal neutral and charged excitations in the confining phase of the associated SU(2) Yang-Mills theory. Due to the nonlocal nature of the latter, a better understanding of nonlocal magnetic correlations of these objects in the plane could, ultimately, affect our understanding of pairing in high-T_c superconductors and other 2D systems involving strongly correlated electrons. Last but not least, making the neutrino a single center-vortex loop subject to an environmentally determined mass could be relevant in interpreting the results of precise measurements of the end-point-spectra for electrons emergent from Tritium beta decays and would have an impact on estimating the (T-dependent) mass of cosmic neutrinos.

References

1. David J. Gross; Frank Wilczek; Ultraviolet Behavior of Non-Abelian Gauge Theories. *Physical Review Letters* **1973**, *30*, 1343-1346, 10.1103/physrevlett.30.1343.
2. H. David Politzer; Reliable Perturbative Results for Strong Interactions?. *Physical Review Letters* **1973**, *30*, 1346-1349, 10.1103/physrevlett.30.1346.
3. Ulrich Herbst; Ralf Hofmann; Emergent Inert Adjoint Scalar Field in SU(2) Yang-Mills Thermodynamics due to Coarse-Grained Topological Fluctuations. *ISRN High Energy Physics* **2012**, *2012*, 1-20, 10.5402/2012/373121.
4. W. Nahm; Self-dual monopoles and calorons. *Exciton Polaritons in Microcavities* **2005**, *201*, 189-200, 10.1007/bfb0016145.
5. Barry J. Harrington; Harvey K. Shepard; Periodic Euclidean solutions and the finite-temperature Yang-Mills gas. *Physical Review D* **1978**, *17*, 2122-2125, 10.1103/physrevd.17.2122.
6. Thomas C. Kraan; Pierre Van Baal; Exact T-duality between calorons and Taub-NUT spaces. *Physics Letters B* **1998**, *428*, 268-276, 10.1016/s0370-2693(98)00411-0.
7. Thomas C Kraan; Pierre Van Baal; Periodic instantons with non-trivial holonomy. *Nuclear Physics B* **1998**, *533*, 627-659, 10.1016/s0550-3213(98)00590-2.
8. Ralf Hofmann; Dariush Kaviani; The Quantum of Action and Finiteness of Radiative Corrections: Deconfining SU(2) Yang-Mills Thermodynamics. *Quantum Matter* **2012**, *1*, 41-52, 10.1166/qm.2012.1004.
9. Dmitri Diakonov; Nikolay Gromov; Victor Petrov; Sergey Slizovskiy; Quantum weights of dyons and of instantons with nontrivial holonomy. *Physical Review D* **2004**, *70*, 036003, 10.1103/physrevd.70.036003.
10. Ralf Hofmann; SU(2) Yang–Mills Theory: Waves, Particles, and Quantum Thermodynamics. *Entropy* **2016**, *18*, 310, 10.3390/e18090310.
11. Niko Krasowski; Ralf Hofmann; One-loop photon–photon scattering in a thermal, deconfining SU(2) Yang–Mills plasma. *Annals of Physics* **2014**, *347*, 287-308, 10.1016/j.aop.2014.04.024.
12. J. Ludescher; R. Hofmann; Thermal photon dispersion law and modified black-body spectra. *Annalen der Physik* **2009**, *18*, 271-280, 10.1002/andp.200910348.
13. Markus Schwarz; Ralf Hofmann; Francesco Giacosa; RADIATIVE CORRECTIONS TO THE PRESSURE AND THE ONE-LOOP POLARIZATION TENSOR OF MASSLESS MODES IN SU(2) YANG–MILLS THERMODYNAMICS. *International Journal of Modern Physics A* **2007**, *22*, 1213-1237, 10.1142/s0217751x07035227.
14. Ralf Hofmann; NONPERTURBATIVE APPROACH TO YANG–MILLS THERMODYNAMICS. *International Journal of Modern Physics A* **2005**, *20*, 4123-4216, 10.1142/s0217751x05023931.
15. R. Hofmann; Low-frequency line temperatures of the CMB (Cosmic Microwave Background). *Annalen der Physik* **2009**, *18*, 634, 10.1002/andp.200910361.
16. Steffen Hahn; Ralf Hofmann; Exact determination of asymptotic CMB temperature-redshift relation. *Modern Physics Letters A* **2018**, *33*, 1850029, 10.1142/s0217732318500293.
17. Planck Collaboration; P. A. R. Ade; N. Aghanim; C. Armitage-Caplan; M. Arnaud; M. Ashdown; Fernando Atrio-Barandela; J. Aumont; C. Baccigalupi; A. J. Banday; et al. Planck2013 results. XVI. Cosmological parameters. *Astronomy & Astrophysics* **2014**, *571*, A16, 10.1051/0004-6361/201321591.
18. Robert H. Becker; Xiaohui Fan; Richard L. White; Michael A. Strauss; Vijay K. Narayanan; Robert H. Lupton; James E. Gunn; James Annis; Neta A. Bahcall; J. Brinkmann; et al. Evidence for Reionization at [ITAL][CLC]z[/CLC][/ITAL] ∼ 6: Detection of a Gunn-Peterson Trough in a [ITAL][CLC]z[/CLC][/ITAL] = 6.28 Quasar. *The Astronomical Journal* **2001**, *122*, 2850-2857, 10.1086/324231.
19. Ralf Hofmann; The fate of statistical isotropy. *Nature Physics* **2013**, *9*, 686-689, 10.1038/nphys2793.
