Time Series Analysis

With the advent of the digital computer, time series analysis has gained wide attention and is being applied to many fields of science. In this survey entry, many traditional and recent techniques for time series analysis and change detection are reviewed, including spectral and wavelet analyses with their advantages and weaknesses. The main focus of the survey is the analysis of unevenly sampled or unequally spaced time series.

1. Introduction

A time series is an ordered sequence of data points measured at discrete time intervals that may not be equally spaced or may contain data gaps, and so the time series is unequally spaced or unevenly sampled. In certain experiments, measurements may also have uncertainties introduced by random noise and thus may also be unequally weighted.

In many areas of research, such as geodesy, geophysics, geodynamics, astronomy, and speech communications, researchers deal with series of data points measured over time or distance to study the periodicity and/or power of certain constituents. For example, in a speech record, one may be interested in certain signals in the record, such as the voice of a person obscured by other simultaneously recorded constituents, noise from musical instruments, and the environment. Furthermore, the physical series of data are usually a combination of various sinusoidal waves, such as light emitted from a star, sound waves, ocean waves, and thus certain phenomena may be studied more accurately by decomposing the data into a new domain, e.g., frequency or time-frequency domains. The decomposition of a series of data points into sinusoidal waves of various frequencies may be imagined as dispersing sunlight to waves of different frequencies representing different colors when it passes through a triangular prism, e.g., a rainbow.

If all statistical properties of a time series, i.e., the mean, variance, all higher-order statistical moments, and auto-correlation function, do not change in time, then the time series is called stationary. A non-stationary time series may contain systematic noise, such as trends, datum shifts, and jumps, indicating that its mean value is not constant in time. In certain fields, such as geodesy, geophysics, and astronomy, a time series is usually associated with a covariance matrix, which means that the time series is stationary or non-stationary in its second statistical moments. In geodynamics applications, seismic noise may contaminate the time series of interest or certain components of the time series may exhibit variable frequency, such as linear, quadratic, exponential, or hyperbolic chirps.

To solve problems regarding heat transfer and vibrations, Joseph Fourier introduced the Fourier series and showed that certain functions can be written as an infinite sum of harmonics or sinusoids. The Fourier transform basically decomposes a time series into the frequency domain using sinusoidal basis functions. The amplitudes of the sinusoids for a set of frequencies generate a spectrum like the light spectrum. Since then, numerous Fourier-based methods have been proposed by researchers for various purposes, such as time series analysis.

2. Related Material

The following open-access article reviews many current and traditional time series analysis methods listed below and briefly mentions their applications in various fields of science and engineering, such as geodesy, geophysics, remote sensing, astronomy, hydrology, finance, and medicine.

Ghaderpour, E.; Pagiatakis, S.D.; Hassan, Q.K. A Survey on Change Detection and Time Series Analysis with Applications. *Appl. Sci.* **2021**, *11*, 6141. DOI: 10.3390/app11136141

Questions discussed in the review paper above include how one may:

1. extract useful information from a time series theoretically and empirically,
2. attenuate noise and regularize time series,
3. detect and classify changes in the time series components, and
4. analyze unequally spaced time series without any interpolations while considering the observational uncertainties.

The reviewed methods with their abbreviations are listed below:

ALFT Anti-Leakage Fourier Transform
ALLSSA Anti-Leakage Least-Squares Spectral Analysis
ASFT Arbitrary Sampled Fourier Transform
BFAST Breaks For Additive Seasonal and Trend
CCDC Continuous Change Detection and Classification
CLSSA Constrained Least-Squares Spectral Analysis
CWT Continuous Wavelet Transform
XWT Cross Wavelet Transform
DBEST Detecting Breakpoints and Estimating Segments in Trend
DFT Discrete Fourier Transform
DTFT Discrete-Time Fourier Transform
DWT DiscreteWavelet Transform
EMD Empirical Mode Decomposition
EWMACD Exponentially Weighted Moving Average Change Detection
EWT EmpiricalWavelet Transform
HHT Hilbert–Huang Transform
IMAP Interpolation by MAtching Pursuit
JUST Jumps Upon Spectrum and Trend
LSCWA Least-Squares Cross Wavelet Analysis
LSSA Least-Squares Spectral Analysis
LSWA Least-Squares Wavelet Analysis
MALLSSA Multichannel Anti-Leakage Least-Squares Spectral Analysis
MIMAP Multichannel Interpolation by MAtching Pursuit
OLS Ordinary Least-Squares
OLS-MOSUM Ordinary Least-Squares Residuals-Based Moving Sum
STFT Short-Time Fourier Transform
WWA Weighted Wavelet Amplitude
WWZ Weighted Wavelet Z-Transform

3. Conclusions

There are many more robust time series analysis techniques proposed by researchers that were not mentioned in this survey. In many practical applications, time series contain seasonal and trend components. Simultaneous estimation of the statistically significant components can provide more accurate and reliable estimates for the time series components and thus are more appropriate for change detection and monitoring. The time series components can also be estimated more accurately when considering the observational uncertainties. Therefore, the observations with higher uncertainties hold less weight during the analysis and vice versa. Computational complexity optimization is another major challenge in time series analysis when dealing with big data sets. An inappropriate algorithm modification for reducing the computational cost can produce unreliable and inaccurate results. Each method presented herein has advantages and weaknesses, and there is plenty of room for researchers and scientists to expand and improve the existing methods.

Finally, we conclude this by the following quote from Nikola Tesla (1856–1943):
If you want to find the secrets of the universe, think in terms of energy, frequency, and vibration. We are just waves in time and space, changing continuously, and the illusion of individuality is produced through the concatenation of the rapidly succeeding phases of existence. What we define as likeness is merely the result of the symmetrical arrangement of molecules that compose our bodies.