Mathematical Descriptions to Study Mangroves

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Mathematical formulations based on empirical knowledge and statistics or dimensional analysis to study the behavior of mangroves, providing information about their flux processes and environmental interactions including the most influential variables. Among these interactions, formulations regarding carbon sequestration, below-ground biomass, and annual mangrove growth can be encountered in literature.

Keywords: carbon storage ; carbon dioxide sequestration ; mangrove ; mathematical model

1. Introduction

Over the years, concern about climate change has grown to the point where industries and any other activity are looking for ways to make themselves more efficient and environmentally friendly. The increase in the Earth's average temperature causes the modification of multiple ecosystems, affecting the species subsisting in it and the effect on the growth of multiple crops due to the carbon dioxide (CO2) concentrations levels in the air [1]. In addition, the increase in the planet's temperature stimulates the melting of permafrost, causing the release of large amounts of trapped CO2 and methane [2]. Trees play an essential role in the capture and retention of CO2, the gas necessary to carry out part of its regeneration, growth, and maintenance processes during its life, having the possibility of increasing their CO2 storage capacity, depending on the conditions of the tree (humidity, CO2 concentration, nutrients, among others) [3].

This study aims to provide information on the equations that have been used recently to study the behavior of trees, specifically mangroves, providing information to other readers about the most relevant works in the determination of stored carbon and the most influential variables. The research focuses on the works presented in the last five years, including some works developed outside this limit, to then select the research focused on the retention of CO2 in mangrove ecosystems, trying to show a better panorama of the variables involved than allowed by more comprehensive assessments of the importance of mangroves.

Trees near the coasts (in direct contact with salt and fresh water) are called mangroves. This kind of tree provides multiple benefits for the development of many species and the humans that live near these coastal areas [4]. These trees have the characteristic of capturing much more CO2 than the species that are in contact with fresh water (much further from the coast) due to the characteristics of the soil, but they do not have the same number of studies due to the lack of accessibility to perform them [4][5]. Mangroves have managed to adapt to salt water and freshwater ecosystems thanks to the morphological adaptations they have developed, such as glands that allow them to expel excess salt, detachment of the seed once mature and ready to settle, and aerial roots for a greater fixation on their muddy soils, among others [4][6].

Mangroves also protect the coasts from strong winds and waves, dissipating the energy generated by their physical characteristics (such as their abundant roots and leaves) [4], also influencing the attenuation of cyclonic winds [7] and carbon fixation both in the soil and its biomass [8][9]. To survive the extreme environmental conditions, they developed unique ecophysiological characteristics [10][11][12][13][14][15][16] for functions such as leveling the salt concentration inside, water flow, and gas exchange [17]. In the following sections, various equations that characterize mangroves are evaluated. Through the evaluation, it was observed that in many cases, each species of tree has characteristics for which the equations developed for that species would not precisely give the same results if they were applied in another species with different configurations.

The text is divided into five main sections; Section 1 contains a brief introduction to the topic, Section 2 presents the methodology used, Section 3 contains the main results, and Section 4 and Section 5 presents the discussions and conclusions of this research.

2. Relevance of Mangroves Modeling to Reap Their Benefits

This section presents the studies found regarding the mathematical modeling of mangroves' characteristics or aspects. Only four aspects regarding mangroves' habilities or benefits were found and are classified as follows: depollution, biomass content, carbon sequestration, and rate of growth.

Regarding depollution, of the two articles reviewed, only one involves mathematical descriptions. The study developed by Ray et al. [18] analyzed the purification properties of the mangrove by performing tests in an empirical model to determine the uptake of vanadium, tantalum, and niobium in the soil, roots, and leaves of eight mangrove species by extracting samples from the Indian Sundarbans in the northeast of the Indian peninsula. Multiple types of equipment were used in the laboratory to transform the samples into data that could be used within the equations observed in their research.

The results indicate that there is a retention of these elements in the different parts of its structure. Such results can be found in the author's article for the different mangrove species analyzed. Error or correlation data were not presented in the study.

There are multiple studies focused on determining the biomass content in tropical forests to know the carbon storage capacity they possess, and with this, the importance they represent in the fight against global warming. Within the studies that focused on the calculation of biomass, there was a certain tendency regarding allometric models that involved measurements in the field for their use, mostly the product of correlations between the variables. Such is the case of Lozano [17], Mohd Zaki et al. [19], among others, who did not use the same equations but had variables in common (see **Table 1**).

Table 1. Allometric equations focused on the determination of aboveground biomass.

Author	Equation		Species	T (°C)	Mean Annual Rainfall (mm y^{-1})
Mohd zaki et al. [20]	$AGB=\exp[[-1.803-0.976\times E+0.967\times \ln(\rho e)+2.673\times \ln(DBH)-0.0299\times[\ln(DBH)2]]]$	**(5)**	**L.t.t.s.**	**22.9–27.7**	**2178**
Da Motta et al. [21]	$AGB=0.251\times \rho e\times DBH2.46$	**(6)**	**L.r.—R.m. A.s.**	**-**	**1320**
Van Vinh et al. [22]	$AGB=0.38363\times DBH2.2348$	**(7)**	**R.a.**	**27**	**1800**
Simpson et al. [23]	$\ln(AGB)=1.63\times \ln(D0)+1.3545\times \ln(H)-2.8853$	**(8)**	**L.r.–A.g. R.m.**	**21–29**	**-**
Chatting et al. [19]	$\log(AGB)=2.14\times \log(CD)+0.20$ $CD=0.3831\times DBH+0.6863$	**(9)**	**A.m.**	**-**	**54**
Kelleway et al. [24]	$AGB=h\times[0.214\times(DBH\times\pi)-0.113]210$	**(10)**	**A.m.**	**-**	**1084**
Clough et al. [25]	$\text{Log }(AGB)=A+(B\times \text{Log}(DBH))$	**(11)**	**R.s.–A.m.**	**35**	**1750**
Prasanna et al. [26]	$V=h6\times(Aba+4\times Am+At)$	**(12)**	**A.m.**	**28–34.2**	**-**
	$Ldb=\sum j=1inj\times Lwj\times Nj$	**(13)**			
	$Bdb=\sum j=1inj\times bwj$	**(14)**			
Makinde et al. [27]	$AGB=e(-3.1141+0.9719 \ln(DBH\times h))$	**(15)**	**T.g.–G.a. I.e.**	**-**	**1850**

With: L.T.T.S. (lowland tropical tree species), L.R. (Laguncularia racemosa), R.a. (Rhizophora apiculata), A.g. (Avicennia germinans), R.m. (Rhizophora mangle), A.s. (Avicennia schaueriana), A.m. (Avicennia marina), R.s. (Rhizophora stylosa), T.g. (Tectona grandis), G.a. (Gmelina arborea), and I.s. (Indigenous species [27]). Source: Own elaboration.

Mohd Zaki et al. [20] presented equations that help reduce uncertainty when estimating carbon stored in forests, including equation (5) (**Table 1**), calculating the aboveground biomass (AGB) using remote sensing and non-linear regression equations in tropical lowland. The study was carried out in Hutan Simpan Ayer Hitam, a Malaysian forest reserve, which had information collected since 2013, and then the researchers carried out another more recent scan and thus completed the information necessary to apply the study. The equation used considers the density of tree species (ρe), as well as the diameter at breast height (DBH). Within the results, there were slightly low values in the determination coefficient (R2= 0.453) between the existence of carbon and the crown projection area, although its correlation turned out to be higher (0.671), attributing the values to the irregularities present in the canopy of the forest being studied.

Da Motta et al. [21] were able to obtain results by using allometric equations such as (6) (**Table 1**), taking measurements in the trees present in an area of 600 m2 and soil samples to identify aspects such as humidity, permeability, and granulometry, among others. This study obtained 2.92 tons of AGB per hectare (t ha−1) and 1.46 t ha−1 of carbon, with a coefficient of determination, for the mangrove species Laguncularia racemosa, which was satisfactory for the author (R2= 0.89). For the determination of AGB, an equation involving the density of the wood and the DBH was used, while for the stored carbon, the literature was used depending on the type and quantity of species found in the analyzed area.

Van Vinh et al. [22] focused their research on mangroves in Southern Vietnam, with the objective of determining the stored biomass through a proposed allometric equation (Equation (7)) that involves DBH, resulting in values between 59.7 and 230.9 mgC ha−1. Additionally, distributions of carbon storage in biomass were assigned: 77.11% for the trunk, 11.87% for the branches, and 11.02% between roots and leaves with a coefficient of determination R2 of 0.976 with a standard error value of 1.17. Simpson et al. [23] worked on the Atlantic coast of Florida, USA to identify the three-year changes concerning carbon storage due to the invasion of mangrove species in marsh areas thanks to Equation (8), using height of the tree in centimeters (H) and the diameter just after the soil surface (D0). Chatting et al. [19] analyzed a mangrove area in Qatar, where the species Avicennia marina was prevalent, generating the Equation (9) used to determine the existing AGB by means of crown diameter (CD) and DBH.

Kusmana et al. [28] conducted destructive tests on 30 tree samples in the mangroves of Cilacap, Indonesia. Kelleway et al. [24] analyzed two mangroves located in marshes in southeastern Australia to quantify changes in mangrove migration to areas corresponding to salt marshes over 70 years. Aerial photographs from different years were used to make comparisons in vegetation, together with field measurements to determine AGB through allometric equations (Equation (10)) using height and DBH. Clough et al. [25] also provided experimental expressions for estimating AGB in mangroves on the north and west coast of Australia (Equation (11)) to determine the differences in the amount of biomass that exists due to environmental variations with DBH and height.

Prasanna et al. [26] applied Equations (12)–(14) in the Karankadu mangrove swamp in southeastern India. These equations were sectioned in a tree to analyze each biomass contribution and then summations were applied, taking into account the base, middle, and top areas of the trunk (Aba, Am, and At, respectively), dry leaf biomass (Ldb), and dry branch biomass (Bdb). A positive correlation was obtained between AGB and DBH (R2= 0.960), but it was not significant with respect to height (R2= 0.349).

Makinde et al. [27] applied geospatial techniques to determine the aboveground and underground biomass through the non-destructive method, interpreting the information provided by said method and applying Equation (15). In **Table 2**, it is possible to observe the allometric equations, mainly focused on the estimation of AGB.

The aforementioned works ([17][20][21][22][23][19][24][25][26][27][28]) used equations that estimate the AGB of a single tree. To determine the AGB of a forest, it will be necessary to carry out an inventory to obtain the measurements and then multiply by the number of trees. In **Table 2**, the parameters most used by the different authors for the determination of AGB can be observed.

Table 2. Parameters used in the equations to determine the AGB.

	Reference								
Parameter	**[20]**	**[21]**	**[22]**	**[23]**	**[19]**	**[24]**	**[25]**	**[26]**	**[27]**
DBH	*	*	*		*	*	*	*	*
D_0				*					
E	*								
ρ_e	*	*	*						
H				*					
CD						*			
V							*		
h								*	*

Source: our elaboration.

Marchio et al. [29] selected two mangrove streams in southwest Florida, with the difference that one of these streams was hydrologically altered by human presence (dredging, channeling, polluted stormwater, compartmentalization of water flows, etc.) to determine the differences between carbon sequestration and the properties of sediments in mangroves once they are disturbed, thanks to Equation (17). As a result, it was possible to show the negative impact in terms of carbon sequestration as a result of the modification of its ecosystem. Neither paper presented had information on the percentage of error or correlation.

Table 3 presents the allometric equations applied to the determination of carbon storage in other sections of the mangrove, as well as an equation focused on the growth of mangroves.

Table 3. Allometric equations focused on other objectives mangrove.

Author	Equation		Species	T (°C)	Mean Annual Rainfall (mm y^{-1})	Objective
Lozano [17]	Cf=K×Ab×[Bi+(Gy×t)]	**(16)**	**R.s.–A.m.**	**26**	**-**	**Carbon sequestration**
Marchio et al. [29]	Cseq=Ad×BD×Cconc	**(17)**	**A.g.–L.r. R.m.**	**23.6**	**1346**	
Chatting et al. [19]	log(BGB)=2.67×log(CD)−0.11	**(18)**	**A.m.**	**-**	**54**	**below ground biomass**
Makinde et al. [27]	BGB=0.2×AGB	**(19)**	**T.g.–G.a. I.e.**	**-**	**1850**	
Rodriguez et al. [6]	AG=Gy×DBH×(1−DBH×HDMax×HMax)274+3×b2−4×b3×DBH2×S×n×te×rel	**(20)**	**R.m.–A.g. L.r.**	**26.6**	**2300**	**Annual mangrove growth**

Among the works focused on growth is Rodríguez et al. A code written in “C” language was used with 16 years of information-gathering through monitoring. An individual-based model (IBM) was applied because it generates quite accurate results, taking into account how variable and complex mangrove ecosystems are according to the author, using Equation (20) (**Table 3**) analyzing three mangrove species: Laguncularia racemosa, Rhizophora mangle, and Avicennia germinans.

Once the model was applied in the selected areas, it was possible to observe how vulnerable the species were to high concentrations of salinity, unlike Avicennia germinans, which continued to grow and produce new seedlings over time. In the end, the model presented the increase in the basal area that would be expected in the following years, varying the concentration of salinity or maintaining them. The work did not present information on percentage of error or correlation.

In order to better understand the equations presented, dimensional analysis was used to verify dimensional homogeneity, as well as the interrelationships of the quantities that compose them. The Rayleigh method was applied to each of the equations as shown below:

AGB=exp[[−1.803−0.976×E+0.967×ln(ρe)+2.673×ln(DBH)−0.0299×[ln(DBH)2]]]

Following the steps of the Rayleigh method, the fundamental dimensions were used:

AGB= α×ρea×DBHb

M=M0×(ML−3)a Lb

where M is for mass,L for length, and t for time.

Then: AGB∝ ρe×DBH3

This procedure was carried out for each of the equations presented in **Table 1** and **Table 3**, but not all of them achieved a consistent result; however, it is to be expected that some expressions do not make physical dimensional sense among the variables they relate to. The results of the dimensional analysis are presented in **Table 4**.

Table 4. Dimensional analysis of equations.

Equation	Parameters	Fundamental Dimensions	Result Using Rayleigh's Method
(5) **(6)**	ρe, DBH	M, L	AGB∝ρe×DBH3
(12)	h,Aba,Am,At	L	V∝ h×Aba×AmAt
(13)	Lwj	M	Ldb∝Lwj
(14)	bwj	M	Bdb∝bwj
(16)	Ab,Bi,t	M, L, t	Cf∝Bi
(17)	Ad,BD,Cconc	M, L, t	Cseq∝ Ad×BD
(19)	AGB	M	BGB∝AGB

3. Conclusions

This investigation presents a critical and systematic review of the aspects considered when modeling mangrove flux processes and environmental interactions. A total of 15 studies were analyzed, where 1 was related to depollution, 11 to biomass content, 1 to a rate of growth, and 2 to carbon sequestration. Each of the models encountered was analyzed regarding mangroves' characteristics (i.e., linearity and similarities).

Among the analyzed studies, it is recommended to use the diameter of the crown instead of the DBH to have more accurate estimates, which represents a quite controversial assumption since most equations found for the determination of aerial biomass use the DBH in their calculations. In this case, it is necessary to consider the level of precision wanted to consider modifying the equation, depending on the diameter of the crown.

The variables necessary for structuring a complete model were identified, from simple measurements such as DBH to the determination of concentrations in the soil, density, and others. It will be necessary to reevaluate the variables that should be obtained, depending on the influence they have on the total value of AGB, in order to be more efficient in terms of equipment and time needed in the modeling processes.

The importance of complying with physical dimensionality lies in the fact that the models can be replicable for different species because they take into account intrinsic characteristics of each species such as its density, for example. Including volume, diameter, and height does not characterize a species. This can be achieved by combining the above with correlation analysis. Looking for relationships based solely on statistics indicates, among several things, that we are not concerned with knowing the complete physical relationship between these two variables.

It is possible to show the importance of studies to estimate CO2 capture and storage, providing information to governments and other researchers to structure new analysis plans based on models, and not by the traditional invasive destruction method, seeking preservation and recovery of mangrove ecosystems while the information regarding these ecosystems continues to increase. In this way, mangrove forests can continue to mitigate the problems of global warming.

References

1. Altieri, M. Cambio climático y agricultura campesina: Impactos y respuestas adaptativas. LEISA Rev. Agroecol. 2009, 14, 5–8.
2. Castro, M. Derretimiento de Los Polos: Evolución, Causas, Consecuencias, Soluciones-Lifeder. Available online: (accessed on 17 July 2020).
3. Food and Agriculture Organization of the United Nations. Carbono Organico del Suelo: El Potencial Oculto; Food & Agriculture Org: Rome, Italy, 2017; ISBN 978-92-5-309681-7.
4. FAO. The World's Mangroves 1980–2005; FAO: Rome, Italy, 2007; ISBN 978-92-5-105856-5.
5. Dat Pham, T.; Xia, J.; Thang Ha, N.; Tien Bui, D.; Nhu Le, N.; Tekeuchi, W. A Review of Remote Sensing Approaches for Monitoring Blue Carbon Ecosystems: Mangroves, Sea Grasses and Salt Marshes during 2010–2018. Sensors 2019, 19, 1933.

6. Rodríguez-Rodríguez, J.A.; Mancera-Pineda, J.E.; Rodríguez-P., J.M. Validación y Aplicación de Un Modelo de Restauración de Manglar Basado En Individuos Para Tres Especies En La Ciénaga Grande de Santa Marta. Caldasia 2016, 38, 285–299.
7. Das, S.; Crépin, A.-S. Mangroves Can Provide Protection against Wind Damage during Storms. Estuar. Coast. Shelf Sci. 2013, 134, 98–107.
8. Mosquera, L.P.; Riaño-Herrera, N.M.; Arcila-Pulgarín, J.; Ponce-Dávila, C.A.; Fotosíntesis, Respiración y Fotorrespiración. 7. Available online: (accessed on 14 May 2020).
9. Solarte, M.E.; Moreno, L.; Melgarejo, L.M., VI; Fotosíntesis y Pigmentos Vegetales. 16. Available online: (accessed on 14 May 2020).
10. Fetcher, N.; Oberbauer, S.F.; Rojas, G.; Strain, B.R. Efectos Del Régimen de Luz Sobre La Fotosíntesis y El Crecimiento En Plántulas de Árboles de Un Bosque Lluvioso Tropical de Costa Rica. Rev. Biol. Trop. 1987, 35, 97–110.
11. Colombo, R.; Marín, O. Relaciones hídricas, Fotosíntesis, y anatomía foliar de dos especies del género Calotropis. Interciencia 2007, 32, 7.
12. Mazda, Y.; Wolanski, E. Hydrodynamics and Modeling of Water Flow in Mangrove Areas, 1st ed.; Elsevier: Amsterdam, The Netherlands, 2009; ISBN 978-0-444-53103-2.
13. Ishida, A.; Yamamura, Y.; Hori, Y. Roles of Leaf Water Potential and Soil-to-Leaf Hydraulic Conductance in Water Use by Understorey Woody Plants. Ecol. Res. 1992, 7, 213–223.
14. Carrasco, O. Transporte de Agua En Plantas. El Balance Hídrico En Tallos de Árboles Nativos Del Norte de Argentina: Uso e Importancia de Los Reservorios Internos de Agua. Ph.D. Thesis, Universidad de Buenos Aires, Buenos Aires, Argentina, 2018.
15. Edwards, W.R.N.; Jarvis, P.G.; Landsberg, J.J.; Talbot, H. A Dynamic Model for Studying Flow of Water in Single Trees. Tree Physiol. 1986, 1, 309–324.
16. Sobrado, M.A. Relation of Water Transport to Leaf Gas Exchange Properties in Three Mangrove Species. Trees 2000, 14, 0258–0262.
17. Torres, Y.L. Los Sumideros De Carbono: Un Anàlisis De La Potencialidad Económica En Un Bosque De Manglar Del Pacìfico Colombiano. Ing. Recur. Nat. Ambiente 2007, 6, 82–92.
18. Ray, R.; Dutta, B.; Mandal, S.K.; González, A.G.; Pokrovsky, O.S.; Jana, T.K. Bioaccumulation of Vanadium (V), Niobium (Nb) and Tantalum (Ta) in Diverse Mangroves of the Indian Sundarbans. Plant Soil 2020, 448, 553–564.
19. Chatting, M.; LeVay, L.; Walton, M.; Skov, M.W.; Kennedy, H.; Wilson, S.; Al-Maslamani, I. Mangrove Carbon Stocks and Biomass Partitioning in an Extreme Environment. Estuar. Coastal Shelf Sci. 2020, 244, 106940.
20. Mohd Zaki, N.A.; Abd Latif, Z.; Suratman, M.N.; Zainal, M.Z. Aboveground Biomass and Carbon Stocks Modelling Using Non-Linear Regression Model. In IOP Conference Series: Earth and Environmental Science; IOP: London, UK, 2016; Volume 37.
21. da Motta Portillo, J.T.; Londe, V.; Moreira, F.W.A. Aboveground Biomass and Carbon Stock Are Related with Soil Humidity in a Mangrove at the Piraquê-Açu River, Southeastern Brazil. J. Coast. Conserv. 2017, 21, 139–144.
22. Vinh, T.V.; Marchand, C.; Linh, T.V.K.; Vinh, D.D.; Allenbach, M. Allometric Models to Estimate Above-Ground Biomass and Carbon Stocks in Rhizophora Apiculata Tropical Managed Mangrove Forests (Southern Viet Nam). For. Ecol. Manag. 2019, 434, 131–141.
23. Simpson, L.T.; Stein, C.M.; Osborne, T.Z.; Feller, I.C. Mangroves Dramatically Increase Carbon Storage after 3 Years of Encroachment. Hydrobiologia 2019, 834, 13–26.
24. Kelleway, J.J.; Saintilan, N.; Macreadie, P.I.; Skilbeck, C.G.; Zawadzki, A.; Ralph, P.J. Seventy Years of Continuous Encroachment Substantially Increases “blue Carbon” Capacity as Mangroves Replace Intertidal Salt Marshes. Glob. Chang. Biol. 2016, 22, 1097–1109.
25. Clough, B.F.; Dixon, P.; Dalhaus, O. Allometric Relationships for Estimating Biomass in Multi-Stemmed Mangrove Trees. Aust. J. Bot. 1997, 45, 1023.
26. Prasanna, J.; Anand, M.; Vijayasekaran, D.; Kumaraguru, A.K. Allometric Model for Estimating above Ground Biomass and Carbon Storage in Karankadu Mangrove Swamp, Palk Bay, Southeast Coast of India. Indian J. Geo-Mar. Sci. 2017, 46, 1682–1692.
27. Makinde, E.O.; Womiloju, A.A.; Ogundeko, M.O. The Geospatial Modelling of Carbon Sequestration in Oluwa Forest, Ondo State, Nigeria. Eur. J. Remote Sens. 2017, 50, 397–413.
28. Kusmana, C.; Hidayat, T.; Tiryana, T.; Rusdiana, O. Istomo Allometric Models for Above- and below-Ground Biomass of Sonneratia Spp. Glob. Ecol. Conserv. 2018, 15, 15.

29. Marchio, D.A.; Savarese, M.; Bovard, B.; Mitsch, W.J. Carbon Sequestration and Sedimentation in Mangrove Swamps Influenced by Hydrogeomorphic Conditions and Urbanization in Southwest Florida. Forests 2016, 7, 116.