Extra Dimensions in Quantum Newtonian Cosmology

Subjects: Quantum Science & Technology

Contributor: Robert Colson Sapp , Jeffery A. Secrest

This entry surveys the role of extra dimensions in Newtonian quantum cosmology, with particular emphasis on large, compactified, and warped dimensional geometries and their impact on the Newtonian potential in the early universe. The discussion begins with a review of Kaluza–Klein type toy models, followed by models with large extra dimensions in which gravity propagates into a higher-dimensional bulk, producing Yukawa-like modifications to the inverse-square law at submillimeter scales. Compactification schemes on toroidal and spherical dimensions are then examined, yielding the spectrum of Kaluza–Klein modes and quantifying their corrections to the Newtonian potential. Warped extra dimensions of the Randall–Sundrum type are also considered, in which a warp factor dimension is introduced; the resulting modifications to the Newtonian interaction in quantum-corrected cosmological settings are discussed in detail.

extra dimensions | Kaluza–Klein theory | warped geometry | brane world | compactification | higher-dimensional gravity | modified Newtonian potential | hierarchy problem | Randall–Sundrum model | bulk spacetime

Over the past century, nature has yielded tantalizing hints that our universe may be made up of more than the three familiar spatial dimensions that are directly accessible by traditional observation. In the early twentieth century, Kaluza and Klein [1][2][3][4][5][6][7] attempted to extend spacetime to five dimensions to unify gravitational and electromagnetic interactions.

Extra dimensions have since been invoked across a wide range of contexts, including

- addressing the hierarchy between the electroweak and Planck scales [8][9][10];
- providing the additional compact dimensions required by string theory [11][12][13][14];
- generating fermion and neutrino mass hierarchies [15] and additional sources of CP violation [16][17];
- supplying new mechanisms for inflation or other early-universe dynamics [18][19][20][21].

Although these additional dimensions are not directly perceived, their presence may be inferred through their influence on physical laws. Klein originally suggested that an extra dimension could be compactified on a circular geometry ($S1$), with a radius on the order of the Planck length—small enough to remain hidden at observable

energies. This idea has been expanded by modern theories, compactifying higher-dimensional spaces on more complex geometries such as tori, spheres, or Calabi–Yau shapes. The momentum along these compactified dimensions becomes quantized, much like a particle in a box. This results in a discrete spectrum of excitations known as Kaluza–Klein (KK) modes [2]. Although KK modes correspond to massive excitations, they are undetectable at current energies because their mass is inversely proportional to the compactification radius. With a radius near the Planck scale, the first KK excitation lies far beyond the reach of present-day accelerators. While these modes are currently beyond direct detection, they offer a distinctive prediction of extra-dimensional models.

Several theories have emerged with the goal of exploring the implications of higher-dimensional spacetimes [4][22][23][24]. Universal Extra Dimensions (UED) models allow Standard Model fields to propagate through all dimensions [25]. Other frameworks, like the Arkani-Hamed, Dimopoulos, Dvali (ADD) model, confine matter fields to a four-dimensional “brane” within a higher-dimensional bulk, allowing only gravity to access the full spacetime [8][26]. Randall–Sundrum (RS) models go further by introducing a warped geometry that localizes gravity near the brane [9][10].

The existence of extra dimensions has profound consequences for both classical and quantum theories of gravity. One of the most direct consequences is a modification to Newton’s inverse-square law at small distances, often expressed through Yukawa-type corrections [27][28][29]. These modifications to the gravitational potential are testable through precision experiments and may provide evidence for—or constraints on—extra-dimensional physics. This entry reviews several representative extra-dimensional frameworks and their gravitational signatures, focusing on how compactification and warping alter the Newtonian potential through Kaluza–Klein modes and short-distance departures from the inverse-square law, with applications to Newtonian quantum cosmology.

References

1. Kaluza, T. Zum Unitätsproblem der Physik. Sitzungsber. Preuss. Akad. Wiss. Berl. 1921, 966, 1–9.
2. Klein, O. Quantentheorie und fünfdimensionale Relativitätstheorie. Z. Phys. 1926, 37, 895–906.
3. Bailin, D.; Love, A. Kaluza–Klein Theories. Rep. Prog. Phys. 1987, 50, 1087–1170.
4. Overduin, J.M.; Wesson, P.S. Kaluza–Klein Gravity. Phys. Rep. 1997, 283, 303–378.
5. Cheng, H.-C.; Feng, J.L.; Matchev, K.T. Kaluza–Klein Dark Matter. Phys. Rev. Lett. 2002, 89, 211301.
6. Hooper, D.; Profumo, S. Dark matter and collider phenomenology of universal extra dimensions. Phys. Rep. 2007, 453, 29–115.
7. Particle Data Group; Workman, R.L.; Burkert, V.D.; Crede, V.; Klempt, E.; Thoma, U.; Tiator, L.; Agashe, K.; Aielli, G.; Allanach, B.C. Review of Particle Physics. Prog. Theor. Exp. Phys. 2022,

2022, 083C01.

8. Arkani-Hamed, N.; Dimopoulos, S.; Dvali, G. The Hierarchy Problem and New Dimensions at a Millimeter. Phys. Lett. B 1998, 429, 263–272.
9. Randall, L.; Sundrum, R. An Alternative to Compactification. Phys. Rev. Lett. 1999, 83, 4690–4693.
10. Randall, L.; Sundrum, R. Large Mass Hierarchy from a Small Extra Dimension. Phys. Rev. Lett. 1999, 83, 3370–3373.
11. Zwiebach, B. A First Course in String Theory; Cambridge University Press: Cambridge, UK, 2009.
12. Polchinski, J. String Theory, Vol. I; Cambridge University Press: Cambridge, UK, 1998.
13. Greene, B. String Theory on Calabi–Yau Manifolds. arXiv 1997, arXiv:hep-th/9702155.
14. Han, T.; Lykken, J.; Zhang, R.J. Kaluza–Klein States from Large Extra Dimensions. Phys. Rev. D 1999, 59, 105006.
15. Grossman, Y.; Neubert, M. Neutrino masses and mixings in non-factorizable geometry. Phys. Lett. B 2000, 474, 361–371.
16. Gherghetta, T.; Pomarol, A. Bulk fields and supersymmetry in a slice of AdS. Nucl. Phys. B 2000, 586, 141–162.
17. Agashe, K.; Perez, G.; Soni, A. Flavor structure of warped extra dimension models. Phys. Rev. D 2005, 71, 016002.
18. Kachru, S.; Kallosh, R.; Linde, A.; Maldacena, J.; McAllister, L.; Trivedi, S. Towards inflation in string theory. J. Cosmol. Astropart. Phys. 2003, 10, 013.
19. Kachru, S.; Kallosh, R.; Linde, A.; Trivedi, S. De Sitter vacua in string theory. Phys. Rev. D 2003, 68, 046005.
20. McAllister, L.; Silverstein, E. String cosmology: A review. Gen. Rel. Grav. 2008, 40, 565–605.
21. Binetruy, P.; Deffayet, C.; Langlois, D. Non-conventional cosmology from a brane-universe. Nucl. Phys. B 2000, 565, 269–287.
22. Csaki, C. TASI Lectures on Extra Dimensions and Branes. arXiv 2004, arXiv:hep-ph/0404096.
23. Quiros, M. Introduction to Extra Dimensions. arXiv 2006, arXiv:hep-ph/0606153.
24. Rizzo, T. Introduction to Extra Dimensions. arXiv 2010, arXiv:1003.1698.
25. Appelquist, T.; Cheng, H.C.; Dobrescu, B.A. Bounds on Universal Extra Dimensions. Phys. Rev. D 2001, 64, 035002.

26. Antoniadis, I.; Arkani-Hamed, N.; Dimopoulos, S.; Dvali, G. New Dimensions at a Millimeter to a Fermi and Superstrings at a TeV. Phys. Lett. B 1998, 436, 257–263.
27. Adelberger, E.G.; Heckel, B.R.; Nelson, A.E. Tests of the Gravitational Inverse-Square Law. Ann. Rev. Nucl. Part. Sci. 2003, 53, 77–121.
28. Hoyle, C.D.; Kapner, D.J.; Heckel, B.R.; Adelberger, E.G.; Gundlach, J.H.; Schmidt, U.; Swanson, H.E. Submillimeter Tests of the Gravitational Inverse-Square Law. Phys. Rev. Lett. 2001, 86, 1418–1421.
29. Kapner, D.J.; Cook, T.S.; Adelberger, E.G.; Gundlach, J.H.; Heckel, B.R.; Hoyle, C.D.; Swanson, H.E. Tests of the Gravitational Inverse-Square Law below the Dark-Energy Length Scale. Phys. Rev. Lett. 2007, 98, 021101.

Retrieved from https://encyclopedia.pub/entry/history/show/132759