Nature of the Bounce in LQC and PQM

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We present a review concerning quantum cosmology in the presence of cut-off physics. We analyze how the Wheeler–DeWitt equation describes the quantum Universe dynamics when a pure metric approach is concerned, showing that the primordial singularity is not removed by the quantum effects. We then analyze the main implications of applying Loop Quantum Gravity prescriptions to the minisuperspace model, i.e., we discuss the basic features of Loop Quantum Cosmology. For the isotropic Universe dynamics, we compare the original m0 scheme and the most commonly accepted formulation, i.e. the so-called ¯m scheme. Furthermore, some fundamental results concerning the Bianchi Universes are discussed. Finally, we consider some relevant criticisms about the real link between the full theory of LQG and its minisuperspace implementation, especially with respect to the preservation of the internal SU(2) symmetry. In the second part of the review, we consider the dynamics of the isotropic Universe and of the Bianchi models in the framework of Polymer Quantum Mechanics. We first address the polymerization in termsof the Ashtekar–Barbero–Immirzi connection and show how the resulting dynamics is isomorphic to the m0 scheme of LQC with a critical energy density of the Universe that depends on the initial conditions for the dynamics. Then we analyze the polymerization of volume-like variables, both for the isotropic and Bianchi I models, and we see that if the Universe volume (the cubed scale factor) is one of the configurational variables, then the resulting dynamics is isomorphic to that for the ¯m scheme of LQC, with the critical energy density value being fixed only by fundamental constants and the Immirzi parameter. Finally, we consider the polymer quantum dynamics of the homogeneous and inhomogeneous Mixmaster model by means of a metric approach. In particular, we compare the results obtained by using the volume variable, which leads to the emergence of a singularity-free and chaos-free cosmology, to the use of the standard Misner variable. In the latter case we deal with the surprising result of a cosmology that is still singular, and its chaotic properties depend on the ratio between the lattice steps for the isotropic and anisotropic variables. We conclude the review with some considerations of the problem of changing variables in polymer cosmology. In particular, on a semiclassical level, we consider how the dynamics can be properly mapped in two different sets of variables (at the price of having to deal with a coordinate dependent lattice step), and we infer some possible implications on the equivalence of the m0 and ¯m scheme of LQC.

Keywords: quantum cosmology ; loop quantum cosmology ; polymer quantum mechanics ; bounce

1. Introduction

The most significant change in the point of view on how to approach the quantization of the gravitational degrees of freedom took place with the formulation of the so-called loop quantum gravity (LQG)[1], especially because this formulation was able to construct a kinematical Hilbert space and to justify spontaneously the emergence of discrete area and volume spectra. LQG relies on the possibility to reduce the gravitational phase space to that of a S U (2) non-Abelian theory[2][3][4][5], and then the quantization scheme is performed by using “smeared” (non-local) variables, such as the holonomy and flux variables, as suggested by the original Wilson loop formulation and by non-Abelian gauge theories on a lattice. The implementation of this new approach to the cosmological setting leads to define the concept of a primordial Big Bounce, already hypothesized in the seventies. However, the cosmological implementation of LQG, commonly dubbed loop quantum cosmology (LQC), has the intrinsic limitation that the basic SU(2)-symmetry underlying the LQG formulation is unavoidably lost.

The effective formulation of LQC is isomorphic to the implementation of polymer quantum mechanics (PQM)[6][7][8][9] to the minisuperspace variables, typically the Universe scale factors. This correspondence allows to investigate some features of the LQC formulation. In this review, we highlight how the evolution of the quantum Universe is sensitive to the considered set of configurational variables: when real connections are polymerized, the resulting picture resembles that which is commonly dubbed the m0 scheme of LQC[10][11][12]; on the other hand, the use of volume coordinates can be associated to the so-called ¯m scenario[13][14][15]. In LQC, the difference in these two schemes is due to the cosmological implementation of the area element as a kinematical or a dynamical quantity (in the ¯m scenario the area gap is rescaled for the momentum variable, i.e., the squared cosmic scale factor).

We will further present a coherent and detailed discussion of the relations existing between LQC and polymer quantum cosmology, also discussing some of the most relevant open questions, especially concerning the equivalence or non-equivalence of the resulting dynamics in different sets of configurational variables.

2. Loop Quantum Cosmology

GR was reformulated as a SU(2) Gauge theory by Ashtekar[16][17] by performing a 3 + 1 splitting of spacetime and using as fundamental conjugate variables a connection and an electric field, which take values in the Lie algebra su(2) of SU(2). Before moving on to quantization, the canonical fields must be appropriately smeared by defining holonomies of the connections alongan edge and fluxes of the electric field across a bidimensional surface S. A key result of the kinematical framework of LQG is the quantization of the geometrical operators of area and volume. In particular, the smallest non-zero eigenvalue of the area operator is a constant quantity depending on fundamental constants and on the Immirzi parameter only; it is called the area gap D, and is a key parameter of the theory.

The dynamics is derived through the implementation of the operators corresponding to the constraints (22); in order to do this, they must first be expressed in terms of the fundamental variables, i.e., holonomies and fluxes, and then quantized, usually through the Dirac procedure[18]. In the construction of the Hamiltonian constraint, a limit appears due to an holonomy around a square that must then be shrinked. However, in LQC, this limit does not exist; the square can be shrinked only until tits area (calculated with respect to the fiducial metric in a kinematical framework) reaches D. The result is the m0 scheme, where the singularity is replaced by a Big Bounce with a critical density depending on initial conditions.

The improved ¯m scheme is developed by introducing the area gap D as a dynamical feature on the physical metric, i.e. rescaled by the squared scale factor. The resulting variable-dependent translational operator is then dealt with by changing to a volume-like variable; this yields a Big Bounce with a critical density that is a universal constant.

Similar schemes can be developed for the anisotropic Bianchi models; in this case, since on a quantum level the Hilbert spaces with and without support on the zero volume eigenvalue decouple, the singularity is avoided in a stronger sense[19][20][21][22].

Over the years, many criticisms have been made on the LQC framework[1][23][24][25][26][27], mainly about the following points: whether the Bounce can be regarded as a semiclassical phenomenon or must be considered a purely quantum effect; the fact that the quantum dynamics is not derived by a symmetry reduction of the full LQG theory, but by quantizing cosmological models that are reduced before quantization; the use of the area gap as a parameter to construct the dynamics of the reduced theory and its effective description.

3. Polymer Cosmology

Polymer Quantum Mechanics (PQM)[6] is a quantization procedure on a lattice that can reproduce effects of cut-off physics. In PQM, one of the variables (usually position) is considered as discrete, making it impossible for the other to be prometed to a well-defined operator; a regularized version is constructed as the incremental ratio of the translational operator on the lattice. On a semiclassical level, this amounts to a formal substitution of the momentum with a sine function dependent on the lattice parameter.

In the isotropic FLRW model expressed through the Ashtekar variables, the implementation of PQM results in a nondiverging behavior of the energy density at the Bounce, thus ensuring the regularization of the singularity, but it is not afixed feature of the dynamics. This result, valid both on a semiclassical and on a quantum level, is very similar to the m0 scheme of LQC. On the other hand, the same model expressed in volume-like variables yields a Bounce with a fixed energy density, as in the ¯m scheme[28][29].

The situation is similar in the Bianchi I model: when using the Ashtekar variables or anisotropic volume-like variables, the Bounce has a dependence on initial conditions, while if the isotropic volume is one of the fundamental variables the energy density at the Bounce is a fundamental constant[30].

On the other hand, in the implementation of PQM on the Mixmaster model, the very presence of the Bounce depends on the chosen discretized variable: when using the logarithmic volume from the Misner variables the model is still singular[31], while if the actual isotropic volume is used the singularity is removed[32]. However, in the former case, the discretization of both the logarithmic volume and the anisotropic Misner variables affects the chaotic behaviour of the classical model: if the lattice step for the logarithmic variable is smaller than that for the anisotropies, chaos usually present near the singularity is tamed.

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