Three Kinds of Butterfly Effects within Lorenz Models

Subjects: Environmental Sciences
Contributor: Bo-Wen Shen, Roger A. Pielke, Xubin Zeng, Jialin Cui, Sara Faghih-Naini, Wei Paxson, Robert Atlas

Within Lorenz models, the three major kinds of butterfly effects (BEs) are the sensitive dependence on initial conditions (SDIC), the ability of a tiny perturbation to create an organized circulation at large distances, and the hypothetical role of small-scale processes in contributing to finite predictability, referred to as the first, second, and third kinds of butterfly effects (BE1, BE2, and BE3), respectively. A well-accepted definition of the butterfly effect is the BE1 with SDIC, which was rediscovered by Lorenz in 1963. In fact, the use of the term "butterfly" appeared in a conference presentation by Lorenz in 1972, when Lorenz introduced the BE2 as the metaphorical butterfly effect. In 2014, the so-called "real butterfly effect", which is based on the features of Lorenz's study in 1969, was introduced as the BE3.

Keywords: butterfly effects ; SDIC ; metaphorical butterfly effect ; Lorenz models ; chaos ; finite predictability

---

Over a span of 50 years, the pioneering study of Lorenz in 1963 (Lorenz, 1963 [1], hereafter referred to as L63) and follow-up studies in 1969 and 1972 (Lorenz, 1969, 1972 [2][3], hereafter referred to as L69 and L72, respectively) have changed our view on the predictability of weather and climate [4] by revealing the so-called sensitive dependence on initial conditions (SDIC), also known as the butterfly effect. A literature review by Ghys in 2015 [5] indicated that the concept of the "butterfly effect" (i.e., SDIC) could be found in studies by Maxwell [6] and Poincare [7][8]. Such a butterfly effect is referred to as the first kind of BE (BE1). In [2], Lorenz applied a metaphor to discuss the possibility of whether a tiny perturbation could eventually create a tornado with a three-dimensional organized, coherent structure. The metaphorical butterfly effect is referred to as the second kind of BE (BE2). The scientific community accepts the idea that butterfly effects were "rediscovered" by Prof. Lorenz. However, one extraordinary contribution by Prof. Lorenz is that his models and methods have provided foundations that have inspired numerous studies and further advanced our understanding of chaotic nature and limited predictability.

Lorenz's studies [1][2][9] laid a foundation for chaos theory, viewed as the third scientific achievement of the 20th century, after relativity and quantum mechanics. After the bestselling book entitled "*Chaos: Making New Science*" by Gleick, 1987 [10], two terms—chaos and the butterfly effect—have been used interchangeably. During the presentation for the 2021 Nobel prize in physics, the pioneering chaos study by Lorenz [1] was cited as a foundation for the awarded studies [11]. A long history exists regarding when and how the term butterfly effect was first introduced. As suggested in Lorenz's book entitled "*The Essence of Chaos*" [9], the term became increasingly noticeable following Lorenz's studies in 1963 and 1972 [1][2]. Over the past five decades, the aforementioned BEs have had direct or indirect impacts on our lives in numerous ways, including in routine numerical weather and climate predictions and even in Hollywood movies such as Jurassic Park. However, the meaning of the term "butterfly effect" in the L63 and L72 studies is not exactly the same. The exact relationship between the original butterfly effect (i.e., BE1 with SDIC) and the metaphorical butterfly effect (BE2) has become controversial. Inaccurate understandings and, thus, interpretations can easily be found. For clarification, this study documents the definitions and major features of the important kinds of butterfly effects.

Over a 10-year period between the L63 and L72 studies, the L69 model with multiscale modes was proposed for revealing the dependence of predictability on various scales (i.e., wavelengths) and suggested better predictability for larger-scale systems [3]. Furthermore, a predictability limit of two weeks has been suggested and highly cited (e.g., [4][12]). The findings regarding a predictability limit of two weeks for the atmosphere in [3] were partly supported (e.g., [13]) or not supported (Smagorinsky, 1969 [14]) by studies conducted near the time of Lorenz's 1969 publication. For example, on pages 291 and 294, Smagorinsky, 1969 [14] documented that "In general even at 21 days, the perturbed map is still far from randomly related to the control—another way of saying that the deterministic limit has not yet been reached". Additionally, Prof. Arakawa believed that the predictability limit is not necessarily a fixed number [12]. Recently, the idea of a predictability limit of two weeks has been challenged by new insights obtained using the L63 and L69 models, as well as generalized Lorenz models (e.g., [15][16][17]; and references therein), and promising simulations using advanced models (e.g., [18][19][20][21]).

As pointed out by Lorenz (e.g., Hillborn, 2004 [22]), the term "butterfly" appeared in Smagorinsky, 1969 [14] earlier than Lorenz, 1972 [2]. Fifty years since [1], Palmer et al., 2014 [4] reanalyzed major features of [3] in order to introduce the so-called "real butterfly effect", terminology invented to emphasize the hypothetical role of small-scale processes in contributing to finite predictability. This butterfly effect is also different from BE1 and BE2 in [1][2]. Here, a comprehensive literature review is provided to illustrate that the meaning of the butterfly effect and its major characteristics within Lorenz's studies have been inadequately interpreted and should be properly demonstrated. Outstanding questions and issues to be addressed include: (1) properly interpreting Lorenz's findings, (2) providing clear definitions for butterfly effects, (3) gaining an understanding of the relationship amongst various BEs, and (4) revealing the influence of butterfly effects on the development of chaos theory, as well as numerical weather and climate models.

To achieve the above goals with the aim of promoting further investigations, below, this study provides definitions for the three major kinds of butterfly effects based on classical Lorenz studies [1][2][3], including the first, second, and third kinds of BE (e.g., BE1, BE2, and BE3), sometimes referred to as L63-, L72-, and L69 BE, respectively.

References

1. Lorenz, E.N. Deterministic nonperiodic flow. J. Atmos. Sci. 1963, 20, 130–141.
2. Lorenz, E.N. Predictability: Does the flap of a butterfly's wings in Brazil set off a tornado in Texas? In Proceedings of the 139th Meeting of AAAS Section on Environmental Sciences, New Approaches to Global Weather, GARP, AAAS, Cambridge, MA, USA, 29 December 1972. 5p.
3. Lorenz, E.N. The predictability of a flow which possesses many scales of motion. Tellus 1969, 21, 289–307.
4. Palmer, T.N.; Doring, A.; Seregin, G. The real butterfly effect. Nonlinearity 2014, 27, R123–R141.
5. Ghys, E. The Butterfly Effect. In Proceedings of the 12th International Congress on Mathematical Education, Seoul, Korea, 8–15 July 2012; Cho, S., Ed.; Springer: Cham, Switzerland, 2012.
6. Maxwell, J.C. Matter and Motion; Dover Publications: Dover, UK, 1952.
7. Poincaré, H. Sur le problème des trois corps et les équations de la dynamique. Acta Math. 1890, 13, 1–270.
8. Poincaré, H. Science et Méthode, Flammarion. English Transl. 1908 ed.; Maitland, F., Ed.; Thomas Nelson and Sons: London, UK, 1914.
9. Lorenz, E.N. The Essence of Chaos; University of Washington Press: Seattle, WA, USA, 1993; 227p.
10. Gleick, J. Chaos: Making a New Science; Penguin: New York, NY, USA, 1987; 360p.
11. The Nobel Committee for Physics. Scientific Background on the Nobel Prize in Physics 2021 "for Groundbreaking Contributions to Our Understanding of Complex Physical Systems". 2021. Available online: https://www.nobelprize.org/prizes/physics/2021/popular-information/ (accessed on 28 June 2022).
12. Lewis, J. Roots of ensemble forecasting. Mon. Weather. Rev. 2005, 133, 1865–1885.
13. Leith, C.E.; Kraichnan, R.H. Predictability of turbulent flows. J. Atmos. Sci. 1972, 29, 1041–1058.
14. Smagorinsky, J. Problems and promises of deterministic extended range forecasting. Bull. Amer. Meteor. Soc. 1969, 50, 286–312.
15. Shen, B.-W.; Pielke, R.A., Sr.; Zeng, X.; Baik, J.-J.; Faghih-Naini, S.; Cui, J.; Atlas, R. Is weather chaotic? Coexistence of chaos and order within a generalized Lorenz model. Bull. Am. Meteorol. Soc. 2021, 2, E148–E158. Available online: https://journals.ametsoc.org/view/journals/bams/102/1/BAMS-D-19-0165.1.xml (accessed on 29 January 2021).
16. Shen, B.-W.; Pielke, R.A., Sr.; Zeng, X.; Baik, J.-J.; Faghih-Naini, S.; Cui, J.; Atlas, R.; Reyes, T.A. Is Weather Chaotic? Coexisting Chaotic and Non-Chaotic Attractors within Lorenz Models. In Proceedings of the 13th Chaos International Conference CHAOS 2020, Florence, Italy, 9–12 June 2020; Skiadas, C.H., Dimotikalis, Y., Eds.; Springer Proceedings in Complexity; Springer: Cham, Switzerland, 2021.
17. Shen, B.-W.; Pielke, R.A., Sr.; Zeng, X. One Saddle Point and Two Types of Sensitivities Within the Lorenz 1963 and 1969 Models. Atmosphere 2022, 13, 753.
18. Shen, B.-W.; Tao, W.-K.; Wu, M.-L. African Easterly Waves in 30-day High-resolution Global Simulations: A Case Study during the 2006 NAMMA Period. Geophys. Res. Lett. 2010, 37, L18803.
19. Shen, B.-W.; Tao, W.-K.; Green, B. Coupling Advanced Modeling and Visualization to Improve High-Impact TropicalWeather Prediction (CAMVis). IEEE Comput. Sci. Eng. 2011, 13, 56–67.
20. Shen, B.-W. On the Predictability of 30-Day Global Mesoscale Simulations of African Easterly Waves during Summer 2006: A View with the Generalized Lorenz Model. Geosciences 2019, 9, 281.

21. Judt, F. Atmospheric Predictability of the Tropics, Middle Latitudes, and Polar Regions Explored through Global Storm-Resolving Simulations. J. Atmos. Sci. 2020, 77, 257–276.
22. Hilborn, R.C. Sea gulls, butterflies, and grasshoppers: A brief history of the butterfly effect in nonlinear dynamics. Am. J. Phys. 2004, 72, 425.