# Mathematical and Logical Realism

Subjects: Others
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According to Balaguer, only two views of the metaphysics of mathematics manage to adequately answer all objections: these are Platonic set theory (as a form of realism) and fictionalism (as a form of anti-realism). Balaguer takes a relatively reserved stance towards the dilemma that one of these two perspectives is correct through three epistemic conclusions: (a) there is no reason to believe or not believe in abstract mathematical objects, (b) it can never in principle have a reason to believe or not believe in abstract mathematical objects, or (c) there is no material fact that could determine between Platonic set theory and fictionalism, although both adequately answer all objections (except, in a way, the question of proving the existence of abstract mathematical objects). The first key question that divides the metaphysics of mathematics into two domains, is the question of whether mathematical theories represent truthful descriptions of some real part of the world. Realistic theories answer affirmatively, while anti-realistic theories answer negatively, claiming that mathematics has no ontology, or that its concepts are objectively empty. The question further divides the realist camp into two groups and is “are mathematical objects spatiotemporal?”. Platonic set theory answers negatively, while anti-Platonism answers affirmatively. The question further divides anti-Platonism into psychologism and physicalism and reads: “what is the nature of mathematical spatiotemporal objects?”. According to psychologism, the objects in question are mental objects, or mathematical statements represent truthful descriptions of mental objects, while according to physicalism, the subject of mathematical statements is non-mental parts of physical reality.

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## 1. Realism

**Anti-Platonism.** John Stuart Mill (1853) [1] argued in the late 19th century that mathematics is an empirical science related to physical objects. According to Mill, abstract objects do not exist, and mathematics tells us nothing about them. The statement “2 + 1 = 3” simply means that when two objects are grouped together and a third is added, a group of three objects results. Psychologism, as another form of anti-Platonism, is divided into two groups: actualist psychologism and possible psychologism. Actualist psychologism sees mathematics as the science of real ideas in people’s heads, while possible psychologism sees it as the science of possible ideas. Although psychologism considers mathematical objects to be products of the subject’s consciousness, it is classified as a form of realism because it acknowledges the ontological content of these objects.

The strongest argument against anti-Platonism was provided by Frege (1953) [2]. It reduces to two statements: (a) the only way to respect the intuition of mathematical theories is to adopt Platonism, and (b) mathematical theories function in practice and are irreplaceable in empirical sciences, therefore they are true—Platonism is accurate and anti-Platonism is inaccurate. This can be answered from the standpoint of fictionalism, that it is meaningless to talk about the truth of the products of fiction.

Several objections can be raised against anti-Platonist physicalism (Millianism), therefore will be discussed in detail.

(a) The poverty of the kingdom of real objects compared to the kingdom of mathematical objects. Millianism claims math is an empirical science, such as sets describing physical sets like Easter eggs. However, sets as mathematical objects pose a problem because an infinite number of sets can have the same physical equivalent. Sets are difficult to reduce to their physical equivalent because each set is its own subset, resulting in an infinite loop.

(b) Overlap of physical equivalents of different mathematical objects in space–time. In the example of an egg, this would mean that infinitely many mathematical objects that can be constructed according to set theory over only one egg actually have one and the same physical equivalent—the egg. Thus, their difference must be based in the non-physical, non-space–time.

(c)

The Empirical Unfalsifiability of Mathematical Statements. If mathematics were a highly generalized empirical science, then its statements would at least in principle be falsifiable by empirical means. However, this does not happen. This means it is not an empirical science.

Psychologism and fictionalism both reject the objective nature of mathematical objects but differ in that psychologism sees mathematics as the study of the ideas of mathematical objects while fictionalism regards them as fiction. Frege [2] argued against psychologism, claiming that it cannot speak about mathematical objects such as the class of real numbers or numbers too large to be conceived. Psychologism also de-objectivizes mathematics as it depends on the existence of consciousness. Psychologism has not been able to address these issues and has been overshadowed by other theories.

**Platonism.** Platonism holds that math is an ontological science of non-physical, non-mental, and non-spatiotemporal objects. Famous proponents include Plato [3], Frege [4], Gödel [5], and Quine [6]. Platonism is divided into traditional and sets-based, with the latter acknowledging an infinite number of mathematical objects. Platonism is also divided into object-based and structuralism, with the latter holding that structures are more important than objects. Balaguer [7] defends sets-based Platonism, while Resnik and Shapiro [8][9] support structuralism (e.g., [10]) to respond to objections such as the non-uniqueness problem and epistemological objection.

The epistemological argument against Platonism can be formulated in several ways, but it most often comes down to the following series of statements:

(1)Human beings are completely space–time-bound.

(2)If there exists any abstract mathematical object, it is necessarily non-spatial–temporal.

(3)If there exists any abstract mathematical object, then human beings cannot have knowledge of it.

(4)If mathematical Platonism is correct, then human beings cannot have mathematical knowledge.

(5)Human beings have mathematical knowledge.

(6)Therefore, mathematical Platonism is not correct.

The key statements are (3) and (5), and they necessarily imply (6). The most common defense against the epistemological argument is that statements (1) and (2) do not imply (3), so even though human beings are completely space–time-bound and mathematical objects are non-spatial–temporal, knowledge of them is possible. The challenge that remains for Platonists in this way is how information is transferred from non-spatial–temporal to space–time. For example, Gödel, as a Platonist, challenged statement (1) by claiming that the human mind is capable of stepping into the non-spatial–temporal domain. Others have challenged (2), claiming that ordinary perception can provide knowledge about abstract mathematical objects, or that they are not non-spatiotemporal in any sense. The third strategy is to accept both (1) and (2) and still attempt to develop some non-contact epistemology based on the flow of information between the two domains, spatiotemporal and non-spatiotemporal. This path has been chosen by the majority of philosophers, including Quine, Resnik, Shapiro, Balaguer, etc. Each mentioned strategy to defend against the epistemological objection has its weaknesses. The pluralism of Platonism abolished the boundary between possibility and existence by claiming that every mathematical object that can exist, exists, and thus, practically enabled each mathematical theory that is consistent to describe some part of the realm of mathematical objects. Its origin, whether it is dreamed, derived, imagined, etc., is completely irrelevant. What is important is that it is consistent, and this alone provides it with ontological content. A series of objections can be raised against this defense, for example, it is difficult to evaluate the consistency of mathematical theories without access to the world of mathematical objects, as well as even if consistent theories describe some part of the realm of mathematical objects. Objectivity could also be challenged, as it is possible to accept two mathematical theories as true even though they are based on two different, incompatible hypotheses, as well as the possibility that following the ontology of pluralism of Platonism leads to contradiction.

In the non-uniqueness objection (b), Platonism is criticized for implying that mathematical theories describe unique sets of mathematical objects (e.g., natural numbers); however, this is not the case. Therefore, Platonism is incorrect. The usual argumentation goes as follows:

(1)If there exists one sequence of abstract mathematical objects that satisfies the rules of Peano arithmetic, then there exists an infinite number of such sequences.

(2)

There is nothing metaphysically special about any of these sequences that would make it the sequence of natural numbers.

(3)It follows that there is no unique sequence of mathematical objects that represents the natural numbers.

(4)Platonism holds that there is a unique sequence of mathematical objects that represents the natural numbers.

(5)Therefore, Platonism is incorrect.

To avoid the non-uniqueness objection, Platonism must either reject (2) by asserting that there is still something metaphysically unique about the sequence of natural numbers or abandon the idea that mathematical theories refer to unique sets of mathematical objects (4). Balaguer [7] argues that statement (4) is mistaken, and that Platonism has never been constituted as a direction that describes unique sets of mathematical objects. According to him, only Platonism of sets, by erasing the boundary between what is and what can be, and accepting the non-uniqueness of mathematical objects described by mathematical theories, deals with the two aforementioned objections and several other minor criticisms.

## 2. Anti-Realism

Anti-realism in mathematics comes in various forms, including conventionalism, formalism, deductivism, meinongianism and fictionalism. Conventionalism asserts that mathematical statements are true by convention, while formalism considers mathematical statements as theorems of the formal system. Deductivism holds that mathematical statements are deductions from basic axioms. Fictionalism regards mathematical objects as fictions, without any reference to the real world. Balaguer [7] criticizes fictionalism’s shallow explanation that mathematics is useful and aesthetically attractive. The debate resembles the materialist–idealist debates on the criterion of truth (see [11]). The challenge is to reconcile the emptiness of mathematics with its practical success.


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