Nonlocal Elasticity for Nanostructures: A Review of Recent Achievements

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Recent developments in modeling and analysis of nanostructures are illustrated and discussed in this paper. Starting with the early theories of nonlocal elastic continua, a thorough investigation of continuum nano-mechanics is provided. Two-phase local/nonlocal models are shown as possible theories to recover consistency of the strain-driven purely integral theory, provided that the mixture parameter is not vanishing. Ground-breaking nonlocal methodologies based on the well-posed stress-driven formulation are shown and commented upon as effective strategies to capture scale-dependent mechanical behaviors. Static and dynamic problems of nanostructures are investigated, ranging from higher-order and curved nanobeams to nanoplates. Geometrically nonlinear problems of small-scale inflected structures undergoing large configuration changes are addressed in the framework of integral elasticity. Nonlocal methodologies for modeling and analysis of structural assemblages as well as of nanobeams laying on nanofoundations are illustrated along with benchmark applicative examples.

Keywords: nonlocal continuum mechanics ; nanostructures ; integral elasticity

According to traditional ideas in continuum mechanics, constitutive equations are those intrinsic relations providing response variables at a material point of a continuum as functions of variables assessed at the same point. Thus, classical constitutive laws support the axiom of local action, stating that response variables at a material point are not affected by the state of the continuum at distant material points. However, in determining the application field of local continuum mechanics, notion of length scale plays a crucial role. Indeed, if a continuum's external characteristic length (i.e., its structural dimension or wavelength) is significantly greater than its internal characteristic length (its interatomic distance or the size of its heterogeneities), then classical constitutive laws can accurately predict the outcome. In contrast, local theories are unable to capture the effective mechanical behavior if the external and internal characteristic lengths are comparable, and nonlocality thus becomes necessary to account for long-range interaction forces. According to nonlocal continuum field theories, constitutive response at a material point of a continuum depends on the state of all points and is thus characterized by response functionals [1][2].

Nowadays, modeling and optimization of smaller and smaller smart devices represent one of the most promising fields of application of nonlocal continuum mechanics due to the growing interest in nanoscience and nanotechnology. The development of mathematical tools able to capture size effects in small-scale structures has been pushed by the increasing attention to miniaturized electromechanical devices, with several potential applications in engineering science. In this regard, the main purpose consists of conceiving effective and computationally efficient methodologies to model size-dependent behavior and design small-scale structures exploiting unconventional tools provided by nonlocal continuum mechanics, rather than time-consuming atomistic approaches [3][4][5].

From the mathematical point of view, nonlocal theories provide enriched constitutive laws that are not pointwise and in which long distance interactions are described by internal characteristic lengths. In [6][7], Eringen developed one of the first theories of nonlocal integral elasticity, according to which stress is the convolution integral between the elastic strain field and a proper averaging kernel governed by an internal characteristic length. Such an integral theory is thus referred to as a strain-driven nonlocal theory, as proposed in [8]. Eringen's constitutive law has been efficiently adopted to solve screw dislocation and surface wave problems, but it turned out to be inconsistent when applied to structural problems due to an incompatibility between the constitutive law and equilibrium condition. Application of the strain-driven nonlocal model to structural mechanics led to alleged paradoxical results, as detected in [9][10] and definitely clarified by [11].

In order to address the issues related to the strain-driven nonlocal theory, several formulations have been conceived in recent years. Among these improved elasticity formulations, two-phase (local and nonlocal) mixture models stand out as a useful tool to overcome the ill-posedness of Eringen's theory and effectively capture scale-dependent mechanical behaviors. A two-phase model based on a convex combination of local and strain-driven integral responses was first proposed by Eringen in [12][13]. The mixture theory of elasticity was then restored in [14][15] to formulate well-posed structural problems, assuming that the local fraction of the two-phase law was not vanishing [11]. An alternative mixture theory to bypass difficulties of the strain-driven purely nonlocal law was proposed in [16], namely the strain difference-based nonlocal model of elasticity.

To account for scale effects in nanostructures, other possible theories assume that constitutive responses depend on both elastic strain fields and higher-order gradient strain fields. Eringen's differential law and the strain gradient model of elasticity were interestingly combined by Aifantis in [17][18][19]. Lim et al. coupled Eringen's nonlocal law with the strain gradient elasticity [20] to address wave propagation problems in unbounded domains, leading to a higher-order differential constitutive equation. In the framework of structural mechanics, the necessity of constitutive boundary conditions associated with the differential formulation of nonlocal gradient elasticity was inferred by Barretta and Marotti de Sciarra in [21][22], where the relevant differential constitutive problem was definitely established. The theory of elastic material surfaces conceived by Gurtin and Murdoch in [23] is another important tool for the modeling of nano-mechanical behaviors. In this framework, a combination of nonlocal integral elasticity and surface elasticity was recently provided in [24] to assess the size-dependent mechanics of nanostructures. Nonlocal mathematical models have also been proposed to capture non-conventional phenomena, such as electric polarization in ferroelectric materials [25], and to address diffusion problems in heterogeneous structures [26].

A total remedy to the issues related to the strain-driven nonlocal elasticity was definitely overcome by the stress-driven integral formulation conceived by Romano and Barretta in [27]. According to this theory, size-dependent mechanical behaviors can be modeled by a new nonlocal elastic law based on a stress-driven formulation that provides a consistent approach inside the integral elasticity framework. The nonlocal elastic strain field at a point of a continuum is given as convolution integral between the local elastic strain and a scalar averaging kernel. The relevant continuum problem is well posed, and size effects due to long range interactions can be effectively captured [28][29][30][31]. In [32], the stress-driven nonlocal elasticity was generalized to a two-phase (local/nonlocal) model that is able to capture both softening and stiffening elastic responses. Moreover, the mixture methodology based on the stress-driven approach is well posed for any local fraction. This theory has been successfully applied in recent contributions, such as in [33][34][35].

Nowadays, growing attention is being paid to modeling and design of ultra-small structural systems exploiting consistent methodologies of nonlocal continuum mechanics. A review on the topic was contributed in [36], in which nano-mechanical behavior is investigated by means of strain-driven-based formulations of nonlocal elasticity. A brief overview can be found in [37], where a collection of works concerning applications of nonlocal theories is provided with a main reference to strain-driven formulations. In the present treatment, a comprehensive overview of new developments and outcomes in the framework of nonlocal continuum mechanics applied to nanostructures is provided. Starting from early formulations of nonlocal mechanics, recent theories of integral elasticity are illustrated and exploited to solve challenging problems of current nanotechnological interest. Innovative nonlocal methodologies to solve complex nanosystems are illustrated. A consistent approach to model nanobeams on nonlocal foundations is finally examined.

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