The Foundation of Classical Mechanics

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Mechanics is the science of the equilibrium and motion of bodies subject to forces. The adjective classical, hence Classical Mechanics , was added in the 20th century to distinguish it from relativistic mechanics which studies motion with speed close to light speed and quantum mechanics which studies motion at a subatomic level.

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Classical mechanics, or more commonly *Mechanics*, is the discipline devoted to the study of the equilibrium and motion of bodies subject to forces; the adjective classical sets it apart from *Relativistic mechanics* which studies motion with speed close to light speed and Quantum mechanics which studies motion at a subatomic level. From a historical perspective *Classical mechanics* is the form of mechanics after Newton; for this reason it is often referred to, though quite improperly, as *Newtonian mechanics*. For the 16th and 17th century one speaks of *Early classical mechanics* while for previous periods the locution *Ancient mechanics* is often used.

What is now called mechanics has played a preponderant role in the development of science since ancient Greece [1]. To explain its influence it is not enough, as done just above, to define mechanics as the reasoned study, that is a science, of the phenomena of equilibrium and motion. First of all, mechanics knows how to measure the phenomena of motion: in other words, however complex their appearances may be, whatever qualitative aspects they reveal, whether it is the changing figure of a cloud, of a waterfall, deformations and resistances of an elastic solid, mechanics knows how to define entirely with the help of numbers these motions, these resistances, these deformations. It is a quantitative discipline.

As for the influence of mechanics on the development of other sciences, it is not difficult to see the reasons for this. First of all, the phenomena of motion or of equilibrium occur constantly and everywhere, whether they appear alone or whether they are accompanied by other more complex phenomena (electrical, chemical, etc.). Mechanics was therefore the necessary basis for other sciences, at least as soon as they wanted to be sufficiently precise.

There was not a historical accident which gave mechanics its preponderance. Among all the phenomena, those which are the least difficult to measure, that is to define completely with numbers, are the phenomena of motion: it is therefore mechanics which became the first of all sciences to take a quantitative form. However, by virtue of the more abstract and geometric nature of the phenomena it studies, it had to vegetate as long as it was not in a condition to assume a quantitative form in all its aspects. Let us follow, for instance, a projectile launched into the air; what will we be able to say precisely about its motion if there is no way to register it? It is therefore understandable that the development of mechanics, or better of that part which studies motion, was at the same time so late and so prodigiously rapid once it had begun, and it immediately took precedence over other sciences, even over those which, like alchemy in the Middle Ages, seemed to precede it ([2], pp. 1–4).

Classical mechanics is today generally considered a mathematical physics discipline and as such of little interest to physics researchers. It is a mathematical theory whose axioms, although they draw inspiration from the physical world, are considered as given; the only developments that can be expected are discoveries of new theorems more or less interesting for the applications of the discipline. It must be said that this point of view is not unassailable and the historical analysis of the discipline provides many elements to counter it.

> Despite the unusually distinguished and successful role Newtonian [classical] mechanics has played in the history of modern science, its foundations have been under vigorous debate since Newton first formulated his laws of motion. Moreover, although the axioms have received much more than two centuries of critical attention from outstanding physicists and philosophers, there still is wide disagreement about what they assert and what their logical status is. The axioms (or their logical equivalents) have been claimed to be either a *priori* truths, which can be asserted with apodictic certainty; [or] to be necessary presuppositions of experimental science though incapable either of demonstration by logic or refutation by observation; or to be empirical generalizations, "collected by induction from phenomena"([3], p. 174).

In the 18th century, a century in which classical mechanics had reached a mature status, many important scientists like Euler and d'Alembert believed that mechanics were an a priori science, that needed no recourse to experience to be established. Otherwise other scientists, such as Daniel Bernoulli for instance, supported the idea of mechanics as an empiric discipline.

The possibility of this long dispute depended on the essence of mechanics. Even those who declared mechanics as an experimental science referred to the experience of everyday life and not to complicated experiments carried out in the laboratory, as was the case, even in the 18th century, for electricity and magnetism or thermology. For instance the law of lever was based on the observation that the more the heavier a body, the more it acts on an arm; the law of motion was based on the constant effect of a cause and that heavy bodies felt downward. The principle of inertia was stated based on simple thought experiments and the principle of action and reaction seemed to be self-evident. Of course no one denied the role of experience when a mechanical theory was be applied. For instance, to evaluate the law of falling of a heavy body, it is not enough to say that its speed varies proportionally to time. It is also necessary to know the value of the constant of proportionality, which is the acceleration of gravity. This can be known only by means of accurate experimentations in a laboratory. Equally the evaluation of masses, forces, times, distances may require sophisticated experiments and instruments.

Up to now no shared formulations of a completely axiomatized formulation of classical mechanics have been provided, even though some attempts have been made [4][5][6][7]. Commonly incomplete axiomatized versions circulate in which in some cases the difference between axioms and theorems is not completely clear. For example, Newton's second law of motion, $f=ma$ can be treated as a principle, a relationship between force f and acceleration $a,$ in which both members of the relationship are considered as primitive terms of the theory, or it can be treated as a simple definition of force. This matter of fact strongly suggests that mechanics needs some more studies by physicists in the attempt to improve its comprehension.

The difference between the two kinds of axiomatic formulations, the complete and the incomplete one, is not only of a quantitative but also epistemological nature. A complete axiomatic theory is by definition a closed system, straightforwardly formalizable with the sufficiently sophisticated language of mathematical logic so that it can contain most of mathematics. It has by definition a hypothetical deductive nature; that is, no truth value is given to the axioms of the theory. What matters is that the theory is coherent, even if this coherence as argued by Gödel's theorems cannot be proved but only assumed in the absence of evident inconsistencies. Primitive terms, definitions, axioms, have no definite meaning in themselves, even if they are given names used in everyday life, such as force, mass, inertial system. These quantities take on values through some rules of correspondence, connecting the real world with the theory, which include all the conceptual difficulties of the physical theory that are avoided in the formalized theory. If the theory and the correspondence rules provide results corresponding to the empirical measurements, the theory is said to be validated (see the next sections).

In incomplete axiomatized theories, one starts from some axioms, generally treated as empirical laws and therefore considered true as such. From these axioms one derives conclusions that are true in themselves because they are deduced by true axioms with the indubitable laws of logic. If it were to be verified that the conclusions were not empirically true, the blame would not be attributed to the theory but to those who applied it, not having been able to take into account the precise manner of the different parameters involved. The incomplete axiomatic approach of contemporary treatments is from a logical point of view perfectly equivalent to the approach of Hellenistic Greece scholars, of Archimedes in particular, as will be clear from the next section. From certain points of view, therefore, it can be said that the essence of mechanics has remained unchanged over a period of more than two thousand years.

There are various formulations of classical mechanics. Perhaps the most important distinction is between a vector formulation that emphasizes the notion of force and refers to Newton and Euler and an analytic formulation that emphasizes the related notions of work and energy and refers to Leibniz and Lagrange. Up to now the equivalence of the two formulations has not been proven in a shared way, even if most scholars are convinced of it, or if they are not convinced they do not consider the fact particularly interesting. It must be said that many textbooks, even of a high level, try to prove this equivalence, unfortunately with some logical leaps. The few supporters of the non-equivalence between analytic and vector theories point out that within the analytic formulation, work and energy are concepts that can be easily derived from thermodynamics, which is not true for the vector formulation.

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