Beyond Special Relativity

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There are two different ways in which one can go beyond the kinematics of Special Relativity (SR). One can consider adding to the Standard Model (SM) Lagrangian new terms that violate Lorentz Invariance (LIV). In case one wants to preserve the relativistic invariance, one should modify the transformations between inertial frames and accordingly modify the special relativistic kinematics; this is what is called Doubly/Deformed Special Relativity (DSR).

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1. Lorentz Invariance Violation

A deviation from SR whose effects increase with the energy can be incorporated in the framework of Effective Field Theory. This is achieved by adding to the fields and symmetries that define the SM of particle physics, terms of dimension higher than four which are not invariant under boosts (neglecting a possible deviation in the rotational symmetry). This is known as the Standard Model Extension (SME) [1][2].

The most important effect of this extension is contained in the free part of the Lagrangian density, i.e., in the part which is quadratic in fields. This leads to a modification of the SR energy–momentum relation of a free particle (modified dispersion relation)

$$E \approx p + \frac{m^2}{2\,p} + \alpha\,\frac{p^{n+1}}{\Lambda^n} \quad \text{when} \quad m \ll p \ll \Lambda\,, \tag{1}$$

where E and p are the energy and the modulus of the momentum of a particle with mass m, respectively, Λ and n are the energy scale and order of correction which parametrize the deviations from SR, respectively, and α is a dimensionless constant which parametrizes the particle's dependence of the LIV effects.

The dimension of the first quadratic term in the SME which violates Lorentz invariance is D=(4+n). There are two alternative values, n=1 (linear case) or n=2 (quadratic case), considered in the studies of LIV. It is also possible to consider a minimal SME [3], where there are only operators of dimension four or less.

A modified dispersion relation implies a modification of the expression of the velocity of a particle in terms of the energy, which can lead to observable consequences from transient astrophysical phenomena (energy-dependent photon time delays), even if the energies of the observed particles are much smaller than the energy scale parametrizing the LIV. Another observable consequence of modified dispersion relations is the modification of the SR kinematics in the different particle processes which are relevant in high-energy astrophysics. The thresholds and the separation of kinematically allowed/forbidden processes (with respect to SR) are affected by the modified energy–momentum relation when the mass-dependent and the LIV terms in (1) become comparable, i.e., when $(m^2/E^2) \sim (E/\Lambda)^n$ [4][5][6]. This happens for $E \ll \Lambda$, and then, one can have observable consequences of the LIV in high-energy astrophysics at energies much lower than the energy scale of LIV.

2. Doubly Special Relativity

It was seen that considering an LIV scenario entails a loss of the relativity principle and the acceptance of a preferred reference frame, which is usually identified with the one defined by the homogeneity and isotropy of the Cosmic Microwave Background (CMB). If one wants to maintain a relativity principle when going beyond SR, one has to consider a deformation of the transformations relating the inertial reference frames. The deformation of SR, usually called DSR [7], is assumed to be parametrized by a new energy scale Λ, which does not usually affect the rotational symmetry, as in the case of LIV.

A necessary ingredient of this departure from SR at the kinematical level is a nontrivial characterization of a multi-particle system with a total energy and momentum differing from the sum of the energies and momenta of the particles. One then has a composition of energy and momentum which is non-symmetric under the exchange of the particles [8]. One arrives at this conclusion from different perspectives of DSR. The starting point of this proposal is the attempt to make compatible the relativistic invariance with the presence of a minimal length [9][10], which seems to be a characteristic of a quantum theory which incorporates consistently the gravitational interaction [11][12][13][14]. Such minimal length can be understood as a consequence of a non-commutativity in a generalization of the classical spacetime, which requires us to go beyond the usual implementation of continuous symmetries by Lie algebras. The new algebraic structure is a Hopf algebra [15] with a non-trivial co-product, which leads to a deformed kinematics with a non-symmetric composition of momenta [16][17]. An alternative way to arrive at the same conclusion is to identify the non-commutativity of spacetime with a non-commutativity of translations in a curved momentum space, which can also be related with to composition of momenta [8][18][19][20][21]. This composition law is therefore a crucial ingredient differentiating DSR and LIV.

Together with the non-linear composition of momenta, the invariance under deformed Lorentz transformations will lead in many cases to a modification of the dispersion relation. As a consequence, in the kinematic analysis of a process in DSR, one has to consider both a possible modification of the energy–momentum relation of the particles participating in it and a modification of the energy–momentum conservation law. The compatibility with the relativity principle, in comparison with the case of LIV, can be shown to produce a cancellation of the effects of the two modifications. Therefore, in order to have an observable consequence of the deformation of the kinematics in a process, one has to consider energies comparable to the energy scale Λ of the deformation [22][23][24][25][26]. This means that in order to have a signal of DSR in the particle processes which are relevant in high-energy astrophysics, it is necessary to consider an energy scale parametrizing the deformation of the kinematics of the order of the energy involved in those processes. At the same time, many of the constraints to the high-energy scale in the case of LIV do not apply in the DSR scenario.

The two previous kinematic ingredients of DSR raise several problems and apparent contradictions on the physical interpretation of the theory. On the one hand, a modification of the composition of momenta in a particle system (independently of the distance between the particles) implies a departure from the notion of absolute locality in spacetime [27][28]. The corresponding loss of the crucial property of cluster [29], which is at the basis of the formulation of special-relativistic quantum field theory, originates the so-called spectator problem [30][31][32]. On the other hand, a modification of the dispersion relation with the associated modification of the velocity of a particle raises an apparent inconsistency of DSR when one applies the deformed kinematics to any system, including a macroscopic system (soccer ball problem [33][34]).

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