Mathematics Problems Solving

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Mathematics problems solving (MPS) has been considered for decades as the centre of mathematics teaching, as it demonstrates the ability to analyse, understand, reason and apply. At the same time, it is also considered to be specific content when highlighting it as a basic competence that students should acquire.

problem solving | emotional development | anxiety | preservice teacher education | mathematics education

1. Introduction

As indicated by [1], different school curricula have established developing and evaluating aspects related to:

(1) The development of the problem: the understanding and analysis of the statement, the design and application of resolution strategies, the verification habits and their coherence within the context presented.
(2) Communicating problem-solving processes and results: the relationship between language and problem solving is specified at different stages around the world in relation to general problem-solving models.
(3) The affective domain and emotional education when assessing personal attitudes, such as perseverance in the search for solutions, confidence in one's ability to achieve them or a positive attitude when it comes to contrasting solutions relative to their peers.

However, these considerations have not been clearly reflected in teaching practice [2]. Thus, students of different educational levels consider MPS as being mechanical and memoristic, they have scarce resources to represent and analyse problems, they do not search for different strategies or methods for their resolution or make use of the different indications made by the teacher [3][4][5][6][7][8][9][10]. Primary education students have internalized that all problems have a single solution that is reached by implementing arithmetic operations using all the data included in the statement. This leads them to develop an automated approach to a known problem format that students follow to reach the solution in a straightforward manner [11]. They also do not perform metacognitive processes, such as generalization and regulation, as well as other processes, such as control and reading [11]. Likewise, in textbooks, the attention to learning heuristic strategies for problem solving [12][13] is conspicuous by its absence.

Regarding communication, students have difficulty verbally explaining the resolution strategy to be developed.

Regarding the affective domain, there have been multiple studies in recent years that have described and analysed the role of affective variables (i.e., beliefs, attitudes, emotions and anxiety) in mathematics and, more specifically,

in MPS. It was found that primary education preservice teachers (PEPSTs) usually have a medium level of anxiety towards mathematics [14][15].

According to Hill et al. [16], math anxiety is a negative emotional response of learners to their current or prospective situation involving mathematics. This happens in an educational process when mathematics teachers introduce new topics with insufficient use of suitable models and a low stage of visualization, and when they present new mathematical notions, such as “ready mathematics”, without steps that show how the new notions were created [17].

Despite the considerable number of research articles on the subject [18] and the proven effectiveness of emotional education in the social, emotional and total self-concept [19], as well as in mathematics learning and problem solving [20], there are no concrete proposals in this regard in mathematics education and, more specifically, in MPS.

Preservice teachers (PEPSTs) expect to be “taught to teach” in mathematics teaching subjects and to be shown “resources to teach mathematics classes” [21], with these expectations being related to their future needs as teachers. Therefore, it is obvious that it is necessary to introduce activities in teacher training curricula so that they can not only extend their knowledge of the mathematical content but also include MPS as content and the affective variables involved in it.

Significant relationships were found between some of the variables related to emotional intelligence and mathematical performance [22][23]. Other research showed the influence of teachers’ affections on those of students and their achievements [24][25].

Considering this situation and the lack of work that has been aimed at intervention in this regard, an emotional and cognitive regulation programme regarding MPS was designed.

The general objective of this programme is to provide future teachers with a didactic tool that allows them to learn and to “learn to teach” in order to manage emotions and self-regulate their learning process in MPS.

To do this, herein aimed to provide PEPSTs with heuristic strategies and techniques for solving mathematical problems; enhancing the search for different “paths” for MPS; and becoming aware of their own conceptions, attitudes and emotions so that they reflect on these variables. By training in techniques for managing behavioural responses towards MPS, they can control and/or modify them.

In this way, the programme intends to develop competences that are related to awareness, regulation and emotional autonomy.

The backbone of the programme is the integrated model of MPS and emotional control (IMMPS, **Figure 1**), which was developed by mathematics authors [26][27] based on emotional education models [20][28][29].

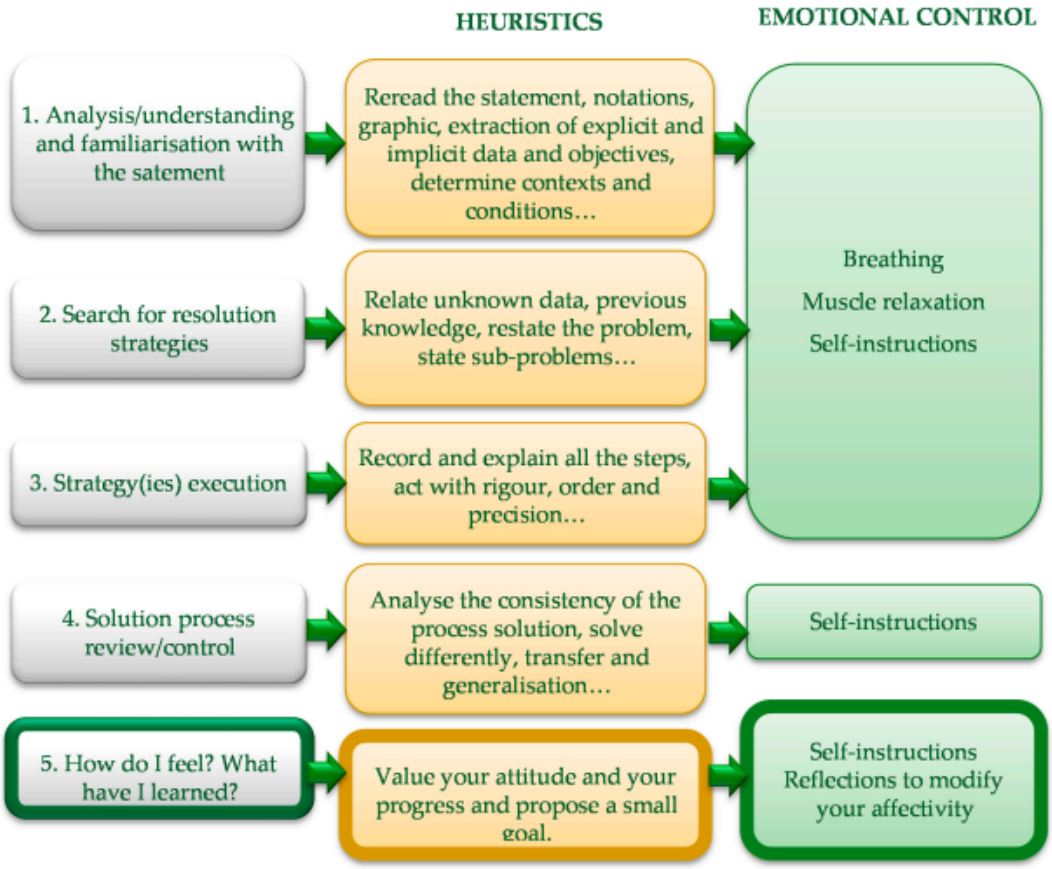

Figure 1. Scheme of the integrated model of MPS and emotional control (IMMPS).

In this way, it combines the heuristics that are used in the different phases of MPS with physiological response management techniques (Jacobson's breathing and muscle relaxation technique) [30] and cognitive techniques (self-instructions) [31].

The IMMPS concludes with a fifth phase that involves personal reflection about (1) the process followed to solve the problem, (2) attitudes and emotions and (3) personal progress, as well as the proposal of small goals for the next mathematical problem to be solved. So far, these aspects have not been considered in other MPS models.

2. Efficacy of an Emotional and Cognitive Regulation Programme for Mathematics Problems Solving

The results found regarding the PEPSTs at the beginning of the programme in relation to anxiety towards MPS coincided with other studies [14][15][32][33][34][35][36][37] in terms of mathematical anxiety in general. It should be noted that the percentage of PEPSTs who presented high levels of anxiety was much higher in the present study. However, these results do not correspond to those carried out with university students, who presented an anxiety level that was lower than the intermediate value of the scale [38].

Students did not develop metacognitive processes, such as generalisation, regulation and process control regarding MPS. They simply reread the statement [11]. The results obtained corroborate what the PEPSTs showed

in a previous study [21] in which learning heuristics was beneficial to be successful in MPS, as was the insistence towards mechanising the MPS process and in teaching methods. The emotional regulation and heuristic techniques presented demonstrated that by including them in the learning process and practising certain mathematical problem-solving methods, the students could learn ways of thinking about approaching and solving problems successfully [17].

Through the implementation of the programme, the PEPSTs achieved reflection, awareness, regulation and emotional autonomy. This resulted in an increase in emotional control, reduced negative emotions and levels of anxiety and improved security and self-confidence as problem solvers and expectations of success.

Similarly, blockages regarding MPS decreased and even disappeared, leading to the PEPSTs developing attitudes that were favourable to MPS such that they persevered in the search for different resolution strategies, including manipulative ones, among other positive attitudes.

On the other hand, the subjects under study dissociated themselves from the traditional conception of a mechanical MPS and the use of formulae, which resulted in the modification of their beliefs about MPS, together with the way it is taught and learnt.

Ultimately, there was a positive evolution relative to the initial self-perception of the PEPSTs as mathematical problems solvers and they showed a greater willingness to initiate changes in MPS.

It is concluded that the relaxation and breathing techniques, self-instructions and a conscious reflection and modification of beliefs about MPS, originating from negative emotions, were facilitators of emotional regulation in this mathematical task. In addition, the IMMPS itself contributed to emotional regulation since it was not the resolution of problems in itself that led to the development of negative emotions but rather not knowing how to act when faced with this task. It showed that teaching a method for teaching problem solving in mathematics and teaching problem-solving strategies through mathematics can develop complex problem-solving skills [17].

The IMMPS had a positive effect on the development of MPS and fostered not only learning, emotional and cognitive regulation but also life skills [17].

In short, the programme achieved positive results, corroborating the importance and need to incorporate emotional skills in the profile of competencies to be developed in the training of future teachers.

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