Teleparallel Equivalent of General Relativity

Subjects: Others
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The teleparallel equivalent of general relativity (TEGR) is an alternative geometrical formulation of the relativistic theory of gravitation. A brief description of the TEGR is presented. The building blocks of the theory and few main achievements are discussed.

Keywords: Teleparallelism ; TEGR ; Gravitational Energy-Momentum ; f(T) theories

1. Introduction

The teleparallel equivalent of general relativity (TEGR) is an alternative geometrical formulation of Einstein's general relativity[1][2]. The TEGR is formulated in terms of the tetrad field and of the corresponding torsion tensor, which is the antisymmetric part of the Weitzenböck connection, in contrast with Einstein's formulation of general relativity, which is constructed out of the metric tensor and of the Riemann-Christoffel curvature tensor. The field equations for the tetrad field in the TEGR are completely equivalent to Einstein's equations for the metric tensor. A theory constructed out of the tetrad field, such as the TEGR, allows the establishment of third rank tensors (the torsion tensor, for instance), and these tensors allow, in turn, the definitions of vector densities and total divergences. These mathematical quantities are suitable for the construction of surface integrals, that are eventually identified with the energy, momentum and 4-angular momentum of the gravitational field. In the metric formulation of general relativity it is not easy to establish non-trivial third rank tensors that yield well defined total divergences of vector fields. Thus, in the TEGR it is possible to define the energy-momentum and 4-angular momentum of the gravitational field. Morever, the definitions of the latter quantities satisfy the algebra of the Poincare group in the phase space of the theory[2][1]. In particular[1], in the TEGR it is possible to establish the definition of the centre of mass moment of the gravitational field. The gravitational centre of mass density is a quantity that describes the regions in space where the gravitational field is more intense, i.e., where geodesic particles acquire stronger gravitational accelerations, compared with the geodesic motion in the flat space-time.

2. Concept of Frame

In the TEGR the concept of frame, determined by the tetrad fields, is of great importance. Observers are adapted to frames in space-time, and the frames are subject to inertial and gravitational accelerations, in general. Inertial accelerations are those that cause the deviation of the motion of free particles, for instance, from the geodesic behaviour. As an example, a static frame is subject to inertial accelerations.

In the geometrical framework of teleparallelism, it is possible to establish the notion of distant parallelism. This feature justifies the name of the geometrical formulation. In order to understand the distant parallelism, one has first to fix a particular frame. In a space–time endowed with a set of tetrad fields, two vectors at distant points in a particular frame are called parallel if they have identical components with respect to the local tetrads at the points considered. It is possible to show[2] that the Weitzenböck connection plays a major role in the establishment of the condition of absolute parallelism in space-time.

All physical features and results that one obtains in the context of Einstein's general relativity are also described in the TEGR. The latter approach to the relativistic theory of gravitation further allows the consideration of additional concepts and definitions, specially regarding the energy-momentum and 4-angular momentum of the gravitational field. Among these applications, we mention the evaluation of the gravitational energy contained within the external event horizon of a Kerr black hole (see[2] and references therein). More recently, the definition of the gravitational centre of mass moment has been applied to the analysis of (non-linear) plane-fronted gravitational waves[3].

Teleparallel theories of gravity are also considered and applied in the investigation of cosmological models[4]. The f(T) models of relativistic gravitation may provide a theoretical interpretation of the late-time universe acceleration, dispensing in this way the cosmological constant, and may easily accommodate with the regular thermal xpanding history, including the radiation and cold dark matter dominated phases.

References

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