Spatial Hurst–Kolmogorov Clustering

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The stochastic analysis in the scale domain (instead of the traditional lag or frequency domains) is introduced as a robust means to identify, model and simulate the Hurst–Kolmogorov (HK) dynamics, ranging from small (fractal) to large scales exhibiting the clustering behavior (else known as the Hurst phenomenon or long-range dependence). The HK clustering is an attribute of a multidimensional (1D, 2D, etc.) spatio-temporal stationary stochastic process with an arbitrary marginal distribution function, and a fractal behavior on small spatio-temporal scales of the dependence structure and a power-type on large scales, yielding a high probability of low- or high-magnitude events to group together in space and time. This behavior is preferably analyzed through the second-order statistics, and in the scale domain, by the stochastic metric of the climacogram, i.e., the variance of the averaged spatio-temporal process vs. spatio-temporal scale.

Keywords: stochastic method ; clustering ; evolution of clustering

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Clustering in nature has been first identified by H.E. Hurst (1951) [1] (Figure 1a) while studying the long-term behaviour in a variety of scales of the discharge timeseries of the Nile River in the framework of developing engineering projects in its basin.

Particularly, H.E. Hurst discovered a tendency of high-discharge years to cluster into high-flow periods, and low-discharge years to cluster into low-flow periods. This behaviour, also known as the Hurst phenomenon or Joseph effect (Mandelbrot, 1977) [2], has been verified in a variety of hydrological [3], hydrometeorological and turbulent processes [4] [5] and in other geophysical and alternate fields such as finance, medicine [6], and art [7] [8] [9].

All these processes are characterized by long-term persistence (LTP), which leads to high unpredictability in long-term scales due to the clustering of events as compared to the purely random process, i.e. white-noise (e.g. as in a fair dice game [10]), or other short-range dependence models (e.g., Markov).

	
(a)	**(b)**

Figure 1: (a) In 1951 H.E. Hurst discovered the clustering behaviour in nature (b) A.N.Kolmogorov proposed a decade before a stochastic process that describes this clustering behaviour.

The mathematical description of the Hurst phenomenon is attributed to A.N. Kolmogorov (Figure 1b) who developed it while studying turbulence in 1940 [11] (Figure 1b), inspiring D. Koutsoyiannis [12] to name the general behaviour of the Hurst phenomenon as Hurst-Kolmogorov (HK) dynamics (Figure 2), to give credit to both contributing scientists and to distinguish it from the Gaussian LTP processes (e.g., fractional-Gaussian-noise [13]), and to incorporate alternate short-range dependence (e.g., Markov-behaviour [14]).

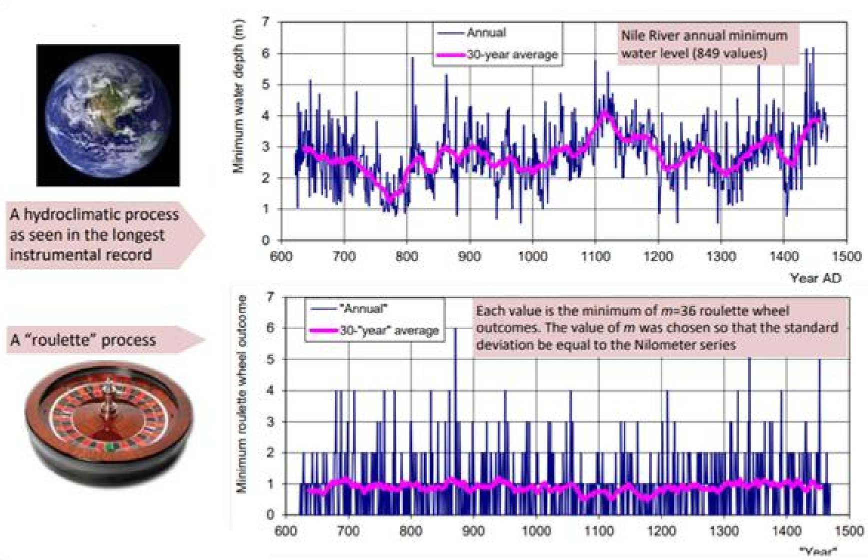

Figure 2. Hurst-Kolmogorov (HK) dynamics and the perpetual change of Earth's climate

The HK dynamics has been recently also linked to the entropy maximization principle, and thus, to robust physical justification [15]. The stochastic simulation of the HK dynamics has been a mathematical challenge since it requires the explicit preservation of high-order moments in a vast range of scales, affecting both the intermittent behaviour in small scales [16] and the dependence in extremes [17] as well as the trends often appearing in geophysical timeseries [18].

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