# Universal Equations for *Wh* and *Wm* Characteristics

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Suter curves for the *Wh* and *Wm* characteristics and four-quadrant (4Q) diagrams of 11 radial pump–turbine models with different specific speeds (*nq* = 24.34, 24.8, 27, 28.6, 38, 41.6, 41.9, 43.83, 50, 56, and 64.04) are presented for the first time, as well as Suter curves for two pump models (*nq* = 25 and 41.8) previously published in the literature. All of these curves were analyzed to establish a certain universal law of behavior, depending on the specific speed.

universal equations for Wh and Wm characteristics
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## 1. Introduction

This research describes a method for obtaining universal equations with all needed formulas, namely, initial, intermediate, and final diagrams of the four-quadrant working characteristics ($Q_{11}$, $n_{11}$, $M_{11}$) and their transition from *H/H*–Q/Q**, *M/M*–Q/Q** diagrams to an *n/n*–Q/Q** diagram, thus giving *H* and *M* curves with various positive and negative percentages, which can then be transformed into Suter diagrams.

As a classical example, the transition from *H/H*–Q/Q**, *M/M*–Q/Q** diagrams to the *n/n** – *Q/Q** diagram is presented, which provides the curves of *H* and *M* with various percentages. The mentioned diagrams were taken from the classical literature [1][2] and then transformed into Suter diagrams. This example considers a pump with *nq* = 25 (35); the original pump is a double-suction pump, where 25 corresponds to the impeller and 35 to the complete machine. Not much data on four-quadrant pump curves can be found in the literature, except for three specific speeds.

A modification of the formulas for re-calculating the four-quadrant characteristics $Q_{11}$, $n_{11}$, $M_{11}$ into Suter curves is made [3][4][5], and the optimal point for the pump–turbine operating mode in pump–turbine models for different *nq* values is defined. The method for re-calculating the four-quadrant (4Q) characteristics $Q_{11}$, $n_{11}$, $M_{11}$ into Suter curves (applied in [5]) is then analyzed. Consequently, for the first time, Suter curves for pump–turbine models (11 models at different specific speeds *nq*), are used to determine the existence of a general law for their form in an attempt to obtain universal curves depending on the specific speed.

For the data analysis and determination of the best-fitting curve, regression and spline methods are utilized.

Experimental research focused on determining the four-quadrant curve characteristics ($Q_{11}$, $n_{11}$, $M_{11}$) carried out in the pump–turbine installations of laboratories in Vienna, Austria, and Wuhan, China are presented. The laboratory in Vienna has one pump–turbine installed, while the laboratory in Wuhan has two pump–turbines. The diagrams obtained according to the measurements are presented in the form of Suter curves for the optimal wicket gate openings (the conducting apparatus) for 11 radial pump–turbine models (which the first author personally collected and re-calculated), together with one Suter curve for the radial pump model from [4].

## 2. Universal Equations for *Wh* and *Wm* Characteristics

The subject is the four-quadrant curves of turbomachines. In order to calculate transient processes in systems with turbomachinery, four-quadrant curves for the machine are required. Researchers mainly consider the curves previously detailed in [2][3][4][6]. In [4], curves are given for only three specific speeds: $nq$ = 25 (35), 147, and 261 (one radial, one semi-axial, and one axial turbomachine).

System designers use the curves most closely describing the analyzed machine, even without interpolation. Of course, many approximations can be used to calculate transient fluid phenomena, almost all of which have already been studied (with respect to unsteady friction, sound speed, fluid–structure interactions, and so on); however, with regard to the wider knowledge that has been obtained, a thorough analysis of various specific speeds (and their impacts) has not yet been published in the literature.

A technique for determining the complete operating characteristics of a hydraulic machine, such as a centrifugal pump or turbine, together on a single diagram has been described in [1]. The characteristics of modern pumps, with high head and high efficiency, were analyzed and presented. The use of these complete characteristics to predict the behavior of the machine during the transient process was discussed and the analytical background was presented. The assumptions involved were explored and experimental checks of their validity were offered. By comparing the possible operating conditions of a hydraulic turbine and a centrifugal pump installation, it soon became clear that pumps are subject to much wider and more involved variations than turbines, especially during the transient states of starting, stopping, or emergency operations.

In [6], the complete characteristics of pumps at certain specific speeds [1800, 7600, and 13,500 (gpm), or 25 (35), 147, and 261 (SI units)] were presented, with the basic test data for the three pumps provided by Prof. Hollander at the California Institute of Technology. A method for creating complete pump characteristics from data obtained during model tests was described. Three sets of pump characteristics were compared, and the effects of certain specific speeds on hydraulic transient processes due to pump disconnection from the network or pump stoppage were determined. The conditions of the transient processes that occur in the radial flow pump, mixed flow pump, and axial flow pump were described. The authors described that, in most cases, complete pump characteristics are not available and incomplete pump characteristics can be extended according to homologous pump laws or similarity laws [5]. The most important connection between a model and a prototype of a pump or turbine is the relations defined by the similarity laws; from these relations, the equations for $Q$, $H$, and $M$ can be derived, which are used during the conversion of data from four-quadrant curves to Suter curves. It is considered that, if the specific speed of the pump under study is approximately the same as the available pump characteristics, the results of the water hammer will be satisfactory for most engineering purposes. Furthermore, although some of the data obtained during the tests will not fit the curves obtained according to the homologous laws, this does not mean that those data are incorrect, but only that the pump during the test did not follow the homologous laws under certain abnormal operations. It was stated that only by using more test data can the average characteristic of the pumps be obtained.

In [3], the authors performed an analysis of transient processes caused by various pump operations, presented a procedure for storing pump characteristics in a digital computer system, developed boundary conditions, and solved a typical problem. They also provide a mathematical representation of the pump's working characteristics. The boundary conditions for load rejection of the pump were explained, as well as the equations for characteristics and the conditions of the boundary, which can be solved simultaneously to determine those conditions. Boundary conditions for more complex cases were developed and, in order to facilitate a better understanding of their implementation, a simple system with only one pump and a very short suction line was considered. A detailed analysis of the procedure for obtaining curves was also provided, indicating the relationships between variables (called pump characteristics). These curves have been presented in various forms suitable for graphical or computer analysis and, of all the methods proposed for storing pump characteristics in a digital computer, the method used by Marchal is considered the most suitable, with some modification. The authors also emphasized that although pump characteristic data in the pumping zone are usually available, relatively few data are available for the dissipation zone or the turbine operation zone.

Explanations related to transient processes and the causes of their occurrence have been provided in [4]. The authors explained that changes in the working state of the turbomachine are the result of non-stationary flow in the hydraulic system, and that this working condition can be caused by starting or stopping the centrifugal pump or by adjusting the load on the generator, which causes changes to occur at that time in the hydraulic turbine. The authors explained how to apply the method of characteristics, using the dimensionless homologous characteristics of turbopumps. They also explained that the conditions for turbines can be described in the same way as for pumps; however, with turbine data, a set of characteristics may be required for each of the many wicket gate openings. The authors specified and analyzed four quantities related to the characteristics total dynamic head, discharge, shaft torque, and rotational speed. They stated two basic assumptions: the characteristics of the equilibrium state hold for the unstable state and, if the discharge and speed rotation change with time, their values at a given time determine the head and torque. They analyzed and explained the homologous relations in detail, noting that homologous theories assume that efficiency does not change with the size of the unit and that it is convenient to work with dimensionless characteristics h, β, v, and α.

In [7], the authors developed a mathematical model that describes the complete characteristics of a centrifugal pump. In this model, a non-linear functional relationship between the parameters of the characteristic operating points (COPs) and the specific speed is established.

In [8], the authors analyzed the complete characteristics of centrifugal pumps and developed a machine learning model to calculate the complete characteristic curve and dimensionless head and torque curves from the quadrant III data set with high accuracy. To obtain a full characteristic curve based on a manufacturer's normal performance curve, the authors developed a machine learning model that predicts full and complete Suter curves using specific pump speeds from known parts of the Suter curve. They used this model to measure and predict the relationship between the data points of quadrant III and those of quadrants I, II, and IV of the centrifugal pump performance characteristic curve.

In [9], the authors analyzed a low-specific speed centrifugal pump with impeller eccentricity based on the N-S equations, and simulated the operation of the pump using the RNG k-e model. The authors studied the change in force induced by the fluid with respect to the eccentricity of the impeller, as well as the non-stationary characteristics of the flow of the internal flow field of the centrifugal pump under different flow conditions and rotation speeds. Furthermore, they performed a detailed analysis of the relationship between the force induced by the fluid of the impeller and the characteristics of the internal flow field.

In [10], the authors simulated a centrifugal pump with a low specific speed under eccentric mounting conditions using the RNG k-e turbulence model. The authors studied the force induced by the fluid of the impeller of a centrifugal pump with a complex vortex. The main part of the research involved investigating the influences of different flow rates, impeller eccentricity, and vortex ratios on the impeller force induced by the fluid.

In [11], based on the basic shapes of the head and power curves in the normal operating zone, the authors described the disadvantages of using a specific speed and presented an improved method for selecting appropriate data from the four quadrants.

In [7], the authors discussed a new method that uses the inherent operating characteristics of a centrifugal pump to predict complete pump characteristics (CPCs). The authors also developed a mathematical model that describes the complete characteristics of a centrifugal pump. They performed measurements for a larger number of CPCs and determined a non-linear functional relationship between the parameters of characteristic operating points (COPs) and specific speeds. By combining a mathematical model with a non-linear relationship, they successfully predicted CPCs for given specific speeds.

In [12], the authors developed a method for fitting a complex pump characteristic curve with large deviations and unevenly spaced data points, using a cubic uniform B-spline. The authors proved that the new method works well when considering the precise construction of data in a wide area into a smooth curve. No disturbances caused by random errors were observed during testing.

In [13], the authors simulated the complete characteristic curve of a reversible pump–turbine based on surface fitting, applying the moving least squares (MLS) approximation for this procedure. The authors also analyzed the influence of the MLS parameters, as well as the following parameters: the coefficient of the weighting function, the number of points in the support domain, and the scale of the radius of the support domain. They also analyzed how these parameters affect the calculation results.

In [14], the authors developed a method for determining three-dimensional internal characteristics based on computational fluid dynamics (CFD) in order to quickly and accurately obtain the Suter curves of double-suction centrifugal pumps. This method includes three-dimensional modeling, setting of pump operating conditions, simulation of the pump flow field, and transformation of the results. The main contribution of this method is that, through the use of CFD technology, the relationships between the flow, speed, head, and torque can be precisely calculated under certain pump operating conditions.

In [15], the authors studied the four-quadrant characteristics of the operating curves of a centrifugal pump, using experimental and numerical approaches during testing. A wide range of centrifugal pump flow rates was analyzed in their CFD calculations. They analyzed the behavior of the pump in four quadrants during transient regimes, and concluded that the results of numerical simulations based on two turbulence models of the equation were in good agreement with the experimental results.

In [16], the authors performed a theoretical and experimental investigation of the transient characteristics of a centrifugal pump in two operating modes: starting and stopping. A numerical model based on the method of characteristics was used to analyze the dynamic characteristics of the pump. The authors concluded that the dynamic characteristics of the pump showed significant deviation from the steady-state characteristics; thus, the developed numerical model could be applied to analyze purely non-stationary cases.

In [17], the authors compared the results obtained by using test equipment and numerical methods for a model of a mixed-flow diffusion pump and concluded that there was extremely good agreement between them. They also confirmed that the behavior of a four-quadrant mixed-flow diffusion pump can be reliably simulated by applying the CFD method. They successfully applied the numerically calculated behavior of the four quadrants during the calculation of the water hammer.

In [18], the authors developed an innovative theoretical approach to predict the flow rate and pressure at the best efficiency point (BEP) for pump and turbine modes based on the principle of conformity of characteristics between the runner and the spiral case. The main contribution was the derivation of a theoretical formula for the characteristics of the impeller in turbine mode. For this derivation, the Euler equation of roto machinery was used, as well as the ratio of speeds at the inlet and outlet of the runner.

In [19], the authors developed an improved method with which the complete characteristics of a centrifugal pump can be obtained. For this method, based on the normal performance curve, a conversion formula of complete characteristics was established. Using this newly developed method, the complete characteristic curves of the centrifugal pump 14SA-10 were obtained.

In [20], the authors developed a method for predicting complete pump curves using only normal operation data, along with curves from other machines with similar specific speeds. To model the complete curves, the authors used a trigonometric series and conducted particle swarm optimization (PSO) for fitting of the coefficients. The authors simulated a pump shutdown, compared the obtained results with the results of laboratory tests, and verified the differences using a modeled curve and a similar curve.

In [21], the authors made a significant contribution by developing a three-dimensional Cartesian coordinate system with relative flow angle, specific velocity, and *Wh* (or *Wm*) value as independent variables, then interpolated these values with a bicubic polynomial to develop a three-dimensional surface visual model for forecasting CPCs.

In [22], the authors developed a new formula for adjusting complete pump characteristics (CPCs) using the least squares method taking into account the specific speed, the relative flow angle, and the characteristic parameters corresponding to the condition of zero flow as independent variables.

In [23], based on an analysis of the full characteristic curve of the pump and using the all-purpose formula and nearest neighbor methods, the authors developed a method for obtaining discrete numerical data of the full characteristic curve of pumps with different specific speeds. The authors also determined that, according to similarity theory, it is theoretically feasible to perform numerical reconstruction of the full pump characteristic curve and that the full characteristic curves of pumps with different specific speeds are very different. For pumps of the same type, the difference between characteristic curves is small, and these curves are very similar after curve numbering.

In [24], the authors performed full testing of the four-quadrant performance considering a hydraulic model of the reactor cooling liquid of an ACP100 pump through a reduced speed test. The obtained data for the four-quadrant curves were complete, and the data were effective for further pump transition processes and the calculation of transient thermal–hydraulic characteristics of the entire cooling system of the reactor.

In [25], the authors developed a method for the prediction of the complete characteristics of a Francis pump–turbine. To develop this method, they used Euler equations and speed triangles on the runner, and obtained a mathematical model that describes the complete characteristics of the Francis pump–turbine. The main contribution was that they combined the developed mathematical model with regression analysis of characteristic operating points (COPs) in order to predict complete characteristic curves for arbitrary specific speeds.

In [26], the authors analyzed how modifications to the runner can have different effects on a centrifugal pump operating in the pump and turbine mode. They used the CFD method to obtain the hydraulic performance of a low-specific speed centrifugal pump operating in both modes and experimentally verified the obtained results. They compared the turbine and the pump and concluded that the pump showed more obvious head variation. The obtained results have great significance for improving the hydraulic performance in both pump and turbine modes, through modification of the geometry of the runner.

In [27], using the finite volume method (ANSIS CFKS) and two-way test equipment, the authors analyzed the unsteady characteristics of the internal flow and the time–frequency characteristics of the pressure fluctuations of the pump as a turbine (PAT) after the shutdown process. The authors made a significant contribution by revealing the characteristics of the transient discharge during the process of a mixed flow pump as a turbine failing. This research is very important for the safe transient operation of a pump as a turbine.

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