Conceptual Advances of Learning Trajectories

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One way to conceptualize professional noticing comes from the three skills described by: (1) identifying the relevant aspects, (2) interpreting the students' understanding, and (3) decision making actions. Regarding the first skill, the teacher identifies significant mathematical elements that students use when solving a given mathematical task (mathematical dimension); in the second skill, the teacher interprets the mathematical understanding of students by connecting the significant mathematical elements, identified in their responses, with cognitive aspects (cognitive dimension); and in the third skill, the teacher uses the interpretation of the students' understanding to decide the actions necessary to improve the teaching process (didactic dimension). Professional noticing can be developed in suitable teaching environments. The recognition of conceptual advances helps to interpret students' thinking and learning trajectories which are effective tools to structure and develop professional noticing.

Keywords: professional noticing ; conceptual advances ; progression model ; learning trajectory

1. Professional Noticing of Students' Mathematical Thinking

Professional noticing is a construct that is used to indicate the act of observing or recognizing relevant events of a situation and acting on them. From this perspective, professional noticing is not an exclusive competence of teaching but is also part of the learning of any profession [1].

Ref. [2] consider that professional noticing as a teaching competence implies identifying and recognizing the relevant aspects in a classroom situation, connecting the identified aspects with the general principles of teaching-learning and applying context knowledge in order to make decisions. Ref. [1] particularized this perspective and conceptualized the professional noticing of students' mathematical thinking as a set of three interrelated skills: identifying relevant mathematical elements in students' responses, interpreting students' mathematical understanding taking into account identified mathematical aspects, and making decisions based on students' thinking to improve their learning.

Several research studies have shown that professional noticing can be developed by using a framework that provides references to teachers (e.g., [3]) and that learning trajectories can provide information to teachers in order to interpret students' understanding (e.g., [4]).

2. Learning Trajectories and Conceptual Advances

Although the learning trajectories have been conceptualized in different ways, they are based on the hypothetical learning trajectories that Ref. [5] presented as part of his model in the mathematics teaching course.

The literature describes the learning trajectories as "predictable sequences of constructs that capture how knowledge progresses from initial levels to more sophisticated levels" [6]. Ref. [7] (p. 83) refer to them as "related and conjectured trajectories through a set of instructional tasks […] to involve children in a progression of development of thinking levels".

Ref. [7] consider that a learning trajectory is composed of a mathematical learning objective, a model of progression in learning a specific domain, and instructional tasks that may support such progression. Objectives are concepts and skills that generate future learning; progression models are levels of thinking, each more sophisticated than the previous one that will lead to the achievement of the goal; and instructional tasks are situations designed to help children learn the ideas and skills necessary to achieve the goal.

Ref. [8] defined the thinking levels of a progression model as successively more complex levels reached by students when they progress in the acquisition of a given mathematical concept. These authors highlighted the importance of the delimitation of thinking levels and, especially, of the reasons that promote changes from one level to another. In this way, a progression model is characterized by the levels of thinking, from now on defined as stages of understanding, and by the conceptual advances that allow the transition of students from a stage to a higher one.

Conceptual advances are fundamental moments in the construction of mathematical structures by students and cause “a change in their ability to think and/or perceive mathematical relationships” [9] (p. 362). This change in the student’s mathematical skills is developed through certain tasks, so that teachers, when observing and comparing the different ways with which students solve them, infer conclusions on the construction of a mathematical concept and on conceptual advances [10].

In this research, the conceptual advances linked to the significant mathematical elements that allow the transition between the stages of understanding in a progression model in a learning trajectory are used to characterize the professional noticing of PPTs.

3. Learning Trajectory of Pattern Generalization

Generalization is a mathematical construct that involves going from the particular to the general and seeing the general in the particular; that is, generalizing consists of universalizing a property observed in a limited number of cases. Specifically, in pattern generalization problems, the first terms of a sequence are presented graphically, numerically, or verbally, and the student must identify a common property in them, generalize that property to all the terms of the sequence (near and far generalizations) and, often, also invert the process (reverse process).

Research focused on the way in which primary students solve generalization tasks of patterns [11] have pointed out the relevant role of understanding three mathematical elements: numerical and spatial structures, functional relationship, and reverse process. The numerical and spatial structures emerge respectively from the number and distribution of the components of each term of the sequence, the functional relationship associates each term of the sequence with its number of components, and the reverse process allows to identify a term of the sequence from its number of components.

These mathematical elements are key to defining the stages of understanding and the conceptual advances of the progression model of a learning trajectory of pattern generalization, since: (1) to continue a sequence, the students must identify a regularity between the spatial and numerical structures, coordinating both structures; (2) to identify a distant term they must establish a functional relationship between the term of the sequence and the number of elements that are part of it; and (3) to identify the term of the sequence from the number of elements that are part of it, they must establish the inverse functional relationship to the previous one, by reversing the process.

From the extension and modification of the stages established by Ref. [12] and Ref. [13], four stages of primary students’ understanding in the learning of pattern generalization have been identified (**Table 1**).

Table 1. Understanding stages of pattern generalization.

Stages	Characterization of Stages
Stage 0	- The student does not generalize. - The student is unable to continue the sequence because he/she does not respect the spatial and/or numerical structure or does not perceive the growing pattern. - The students do not coordinate spatial and numerical structures, preventing them from progressing.
Stage 1	- The student performs a near generalization. - The student is able to continue the sequence for near terms because he/she identifies the growing pattern by coordinating spatial and numerical structures. - The students do not relate the term of the figure with its number of elements, which prevents them from generalizing far terms.

Stages	Characterization of Stages
Stage 2	- The student makes a far generalization. - The students are able to continue the sequence for far terms because they identify the functional relationship between the figure term and its number of elements, and they are able to establish a general rule to find the number of elements of any given element. - The students do not identify the inverse functional relationship, necessary to find out an element of the sequence from the number of elements, which prevents them from reversing the process.
Stage 3	- The student reverses the process. - The students are able to identify the inverse functional relationship, allowing them to find any term in the sequence from its elements.

When moving from Stage 0 to Stage 1, the student needs to coordinate the spatial and numerical structures to find the growth pattern; to move from Stage 1 to Stage 2, they need to establish the functional relationship between the term of the sequence and the number of its elements, to find the number of elements of any term; and to move from Stage 2 to Stage 3, they need to reverse the process to find any term of the sequence from its elements. Three conceptual advances linked to the mathematical elements that allow the transition between the stages of understanding are thus determined: the coordination between spatial and numerical structures, the recognition of the functional relationship, and the reversibility of the process.

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