Role of Visual Tools in Understanding Mathematical Culture

Subjects: Mathematics
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The term mathematical culture' cannot be naturally defined; we will understand it in the same way as 'good mathematics' is understood by, i.e., good ways of solving mathematical issues, good mathematical techniques, good mathematical applications and cultivating of mathematical insight, creativity and beauty of mathematics. Cultivation of a mathematical culture means teaching how to see the roots of mathematics in reality (in nature, in society, but also in mathematics itself), getting to know the world of mathematical concepts, understanding this world and being able to apply it in a cultivated and correct way when solving various problems.

mathematical culture | mathematical proof | graph theory | visual tool | teaching

1. Introduction

Developing a mathematical culture means developing the ability to get oriented in mathematical concepts, to understand their essence and to see them and to be able to use them in mutual contexts. That means, to develop a mathematical way of thinking by using abstraction and generalization.

Mathematics as a school subject has long-term beneficial effects on the life of students, supporting long-term learning [1] and positively impacting their socioeconomic status [2]. It is, thus, of essence to promote increasingly efficient means of understanding mathematical concepts to perpetuate mathematical knowledge and thinking in an inclusive manner, helping those with weaker mathematical abilities [3] and promoting discovery learning [4] using digital tools to visualise mathematical proofs.

2. Mathematical Proof

Mathematical proof has been considered an essential part of mathematical science. The strictly defined notion of mathematical proof clearly separates mathematics from the spectrum of other scientific disciplines. Mathematical proof, unlike proofs in other fields, is fundamentally unquestionable. [5]

A mathematical proof can be understood and defined in the following ways:

- 'A mathematical theorem is a proposition, a mathematical proof is clearly in some sense a collection or pattern of propositions' [6].

- Kitcher [7] gave the following definition:

  A proof can be defined as a sequence of statements such that every member of the sequence is either a basic a priori statement or a statement which follows from previous members of the sequence in accordance with some apriority-preserving rule of inference.

- Hanna [8] (p. 6) claimed:

  A formal proof of a given sentence is a finite sequence of sentences such that the first sentence is an axiom, each of the following sentences is either an axiom or has been derived from preceding sentences by applying rules of inference, and the last sentence is the one to be proved.

- ‘A proof of a mathematical theorem is a sequence of steps which leads to the desired conclusion’ [9].

3. Graph Theory Education

Biggs et al. [10] (p. 1) described the origins of graph theory as follow:

The origins of graph theory are humble, even frivolous. Whereas many branches of mathematics were motivated by fundamental problems of calculation, motion and measurement, the problems which led to the development of graph theory were often a little more than puzzles, designed to test ingenuity rather than stimulate the imagination. However, despite the apparent triviality of such puzzles, they captured the interest of mathematicians, the result of which being that graph theory has become a subject rich in theoretical results of surprising variety and depth.

Teaching methods are always a subject of extensive discussions. Teachers look for different ways how to clarify a particular issue to students, see, e.g., recent papers by Morris and Rohs [11], Fussell and Truong [12] and Yohannes and Chen [13].

Demonstration and visualization make the subject much clearer and more comprehensible [14][15][16][17][18]. Teaching/learning model using suitable visual tools represents one of the sustainable trends to develop, pay attention and interest to [19].

In subjects dealing with graph theory there is no problem in illustrating the needed concepts and algorithms using graphs. However, it is very important to prepare suitable illustrative graphs and also to have a tool which can highlight and emphasize properties of and relationships between the discussed concepts.

4. Visual Represantion of Proof

To enable students to come to in-depth understanding of a given problem and understanding of processes that solve the problem, it is necessary to pay close attention to the theory related to the problem, including a thorough analysis of the relevant theorems and their proofs. Senk claims that ‘proof writing has been perceived as one of the most difficult topics for students to learn’ [20] (p. 448), and our experience has made us fully agree with this opinion. The issue of proving theorems is one of the scariest topics for our students. Proofs are perceived as a highly abstract issue, and students attending the Graph Theory course have expressed their demands for better visual presentation of the proofs so that they can better and more deeply understand their concept.

The proof visualization, or in other words “proof without words”, appeared already in the deep past. A nice example of a static visual proof and also of evidence of a high level of mathematical culture achieved by the Greek civilization can be seen, for example, in the article The Binomial Theorem and a Silver Stater from Aegina [21].

New technology development and subsequent introduction of computers into our learning environment resulted in creating new applications, visualization software and multimedia interactive tools enabling dynamic visualizations. Good experience with the GrAlg program and knowledge of the fact that ‘pictures often play a crucial role in logical demonstration’ [22] (p. 1257), resulted in the researchers' idea to create a similar visual tool to help students within the Graph Theory course to better understand the abstraction of the discussed process of mathematical proofs.

The impact and the effectiveness of visualization in teaching mathematical proofs has been of interest to many experts for numerous years. However, in the sphere of graph theory there is hardly any evidence of such a kind of research. This effect has been investigated mostly in geometry. The reason for this situation can be the existence of several kinds of dynamic geometry software suitable for conducting proofs in this field. A few pedagogical experiments related to teaching geometry are mentioned below.

Marrades and Gutierez [23] dealt with teaching geometry at high schools, and investigated whether the use of dynamic geometry could facilitate students’ transition from experimental work with mathematical objects to formal deductive proof. Their study found that teaching geometry in a dynamic computer environment helped students to better understand the abstraction of the rationale of the discovered relationships.

Gawlick [24] examined the effect of dynamic geometry on discovering, testing and proving hypotheses, and, on one hand, he found that students working with dynamic geometry did not perform better in terms of knowledge and skills, but, on the other hand, they were more successful in solving tasks on discovering relationships between geometric objects. He claimed that dynamic geometry supported developing hypotheses, but not their verification.

Baccaglini-Frank and Mariotti [25] investigated students’ cognitive processes in solving open problems in the form of construction problems using dynamic geometry. Based on observations of students’ work and on interviews with them, they concluded that dynamic geometry supported inductive ways of thinking and developed students´ specific form of argumentation.

Erbas and Aydogan-Yenmez [26] investigated the effects of using the DGE dynamic geometry environment with sixth-grade students. The performance of the students in the experimental group, who used DGE in geometry lessons, was better than the performance of the students in the control group, who did not use that environment. In addition, the students in the experimental group showed more interest in the subject matter, and their comments on and their interpretations of the problem were more accurate.

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